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Growth of face-homogeneous tessellations*

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Abstract: A tessellation of the plane is *face-homogeneous* if for some integer $k \geq 3$ there exists a cyclic sequence $\sigma = [p_0, p_1, \ldots, p_{k-1}]$ of integers ≥ 3 such that, for every face f of the tessellation, the valences of the vertices incident with f are given by the terms of σ in either clockwise or counter-clockwise order. When a given cyclic sequence σ is realizable in this way, it may determine a unique tessellation (up to isomorphism), in which case σ is called *monomorphic*, or it may be the valence sequence of two or more non-isomorphic tessellations (*polymorphic*). A tessellation whose faces are uniformly bounded in the hyperbolic plane but not uniformly bounded in the Euclidean plane is called a *hyperbolic tessellation*. Such tessellations are well-known to have exponential growth. We seek the face-homogeneous hyperbolic tessellation(s) of slowest growth rate and show that the least growth rate of such monomorphic tessellations is the "golden mean," $\gamma = (1 + \sqrt{5})/2$, attained by the sequences $[4, 6, 14]$ and $[3, 4, 7, 4]$. A polymorphic sequence may yield non-isomorphic tessellations with different growth rates. However, all such tessellations found thus far grow at rates greater than γ.

Keywords: Face-homogeneous, tessellation, growth rate, valence sequence, exponential growth, transition matrix, Bilinski diagram, hyperbolic plane.

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Rast lično homogenih tlakovanj*

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Povzetek: Tlakovanje ravnine je *lično homogeno*, če za neko celo število $k \geq 3$ obstaja takšno ciklično zaporedje $\sigma = [p_0, p_1, \ldots, p_{k-1}]$ celih števil ≥ 3, da so za vsako lice f tlakovanja stopnje vozlišč incidentnih z f dane s členi σ v bodisi smeri kazalcev na uri bodisi v nasprotni smeri. Kadar je dano ciklično zaporedje σ mogoče predstaviti na ta način, lahko določa bodisi enolično tlakovanje (do izomorfizma natančno), in v tem primeru se σ imenuje *monomorfno*, bodisi je to zaporedje stopenj dveh ali več neizomorfnih tlakovanj (*polimorfno*). Tlakovanje, katerega lica so enakomerno omejena v hiperbolični ravnini, ne pa v evklidski ravnini, se imenuje *hiperbolično tlakovanje*. Za takšna tlakovanja je znano, da imajo eksponentno rast. Iščemo lično homogena hiperbolična tlakovanja najmanjše stopnje rasti, in pokažemo, da je najmanjša stopnja rasti takšnih monomorfnoh tlakovanj "zlati rez," $\gamma = (1+\sqrt{5})/2$, dobljen z zaporedji $[4, 6, 14]$ in $[3, 4, 7, 4]$. Polimorfno zaporedje lahko ustreza neizomorfnim tlakovanjem z različnimi stopnjami rasti. Vendar pa vsa takšna tlakovanja, najdena doslej, rastejo hitreje kot γ.

Ključne besede: Lično homogen, tlakovanje, stopnja rasti, zaporedje stopenj, eksponentna rast, prehodna matrika, diagram Bilinskega, hiperbolična ravnina.

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