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# Finite actions on the 2-sphere, the projective plane and I-bundles over the projective plane

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**Abstract:** In this paper, we consider the finite groups which act on the 2-sphere $\mathbb{S}^2$ and the projective plane $\mathbb{P}^2$, and show how to visualize these actions which are explicitly defined. We obtain their quotient types by distinguishing a fundamental domain for each action and identifying its boundary. If $G$ is an action on $\mathbb{P}^2$, then $G$ is isomorphic to one of the following groups: $\mathbb{S}_4$, $\mathbb{A}_5$, $\mathbb{A}_4$, $\mathbb{Z}_m$ or $\mathrm{Dih}(\mathbb{Z}_m)$. For each group, there is only one equivalence class (conjugation), and $G$ leaves an orientation reversing loop invariant if and only if $G$ is isomorphic to either $\mathbb{Z}_m$ or $\mathrm{Dih}(\mathbb{Z}_m)$. Using these preliminary results, we classify and enumerate the finite groups, up to equivalence, which act on $\mathbb{P}^2 \times I$ and the twisted I-bundle over $\mathbb{P}^2$. As an example, if $m > 2$ is an even integer and $m/2$ is odd, there are three equivalence classes of orientation reversing $\mathrm{Dih}(\mathbb{Z}_m)$-actions on the twisted I-bundle over $\mathbb{P}^2$. However if $m/2$ is even, then there are two equivalence classes.

**Keywords:** Achiral symmetry, chiral symmetry, equivalence of actions, finite group action, isometry, orbifold, symmetry.

Math. Subj. Class.: 57S25, 05E18, 57M60, 57R18, 58D19, 57M20

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# Končna delovanja na 2-sferi, projektivni ravnini in I-svežnjih nad projektivno ravnino

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**Povzetek:** V tem članku obravnavamo končne grupe, ki delujejo na 2-sferi $\mathbb{S}^2$ in projektivni ravnini $\mathbb{P}^2$, in pokažemo, kako vizualizirati ta delovanja, ki so eksplicitno definirana. Dobimo njihove kvocientne tipe z razlikovanjem fundamentalne domene za vsako delovanje in identificiranjem njenega roba. Če $G$ deluje na $\mathbb{P}^2$, potem je $G$ izomorfna eni od naslednjih grup: $\mathbb{S}_4$, $\mathbb{A}_5$, $\mathbb{A}_4$, $\mathbb{Z}_m$ ali $\mathrm{Dih}(\mathbb{Z}_m)$. Za vsako grupo obstaja samo en ekvivalenčni razred (za konjugiranje) in pri delovanju $G$ je zanka, ki obrne orientacijo, invariantna, če in samo če je $G$ izomorfna bodisi $\mathbb{Z}_m$ bodisi $\mathrm{Dih}(\mathbb{Z}_m)$. Z uporabo teh preliminarnih rezultatov klasificiramo in enumeriramo, do ekvivalence natančno, končne grupe, ki delujejo na $\mathbb{P}^2 \times I$ in zasukanem I-svežnju nad $\mathbb{P}^2$. Na primer, če je $m > 2$ sodo celo število in je $m/2$ lih, potem obstajajo na zasukanem I-svežnju nad $\mathbb{P}^2$ trije ekvivalenčni razredi $\mathrm{Dih}(\mathbb{Z}_m)$-delovanj, ki obrnejo orientacijo. Če pa je $m/2$ sod, potem sta takšna ekvivalenčna razreda le dva.

**Ključne besede:** Akiralna simetrija, kiralna simetrija, ekvivalenca delovanj, končno delovanje grup, izometrija, orbifold, simetrija.

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