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Classification of convex polyhedra by their rotational orbit Euler characteristic

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Abstract: Let \mathcal{P} be a polyhedron whose boundary consists of flat polygonal faces on some compact surface $S(\mathcal{P})$ (not necessarily homeomorphic to the sphere S^2). Let $vo_R(\mathcal{P})$, $eo_R(\mathcal{P})$, $fo_R(\mathcal{P})$ be the numbers of rotational orbits of vertices, edges and faces, respectively, determined by the group $G = G_R(\mathcal{P})$ of all the rotations of the Euclidean space E^3 preserving \mathcal{P} . We define the *rotational orbit Euler characteristic* of \mathcal{P} as the number $Eo_R(\mathcal{P}) = vo_R(\mathcal{P}) - eo_R(\mathcal{P}) + fo_R(\mathcal{P})$.

Using the Burnside lemma we obtain the lower and the upper bound for $Eo_R(\mathcal{P})$ in terms of the genus of the surface $S(\mathcal{P})$. We prove that $Eo_R \in \{2, 1, 0, -1\}$ for any convex polyhedron \mathcal{P} . In the non-convex case Eo_R may be arbitrarily large or small.

Keywords: Polyhedron, rotational orbit, Euler characteristic.

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Klasifikacija konveksnih poliedrov glede na njihovo Eulerjevo karakteristiko rotacijskih orbit

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Povzetek: Naj bo \mathcal{P} polieder, katerega površje sestoji iz ploskih poligonskih lic na neki kompaktni ploskvi $S(\mathcal{P})$ (ne nujno homeomorfnii sferi S^2). Naj bodo $vo_R(\mathcal{P}), eo_R(\mathcal{P}), fo_R(\mathcal{P})$ števila rotacijskih orbit vozlišč, povezav in lic, določena z grupo $G = G_R(\mathcal{P})$ vseh takšnih rotacij evklidskega prostora E^3 , ki ohranjajo polieder \mathcal{P} . Definiramo *Eulerjevo karakteristiko rotacijskih orbit* poliedra \mathcal{P} kot število $Eo_R(\mathcal{P}) = vo_R(\mathcal{P}) - eo_R(\mathcal{P}) + fo_R(\mathcal{P})$.

S pomočjo Burnsidove leme dobimo spodnjo in zgornjo mejo za $Eo_R(\mathcal{P})$, ki ju izrazimo kot funkcijo reda ploskve $S(\mathcal{P})$. Dokažemo, da je $Eo_R \in \{2, 1, 0, -1\}$ za vsak konveksen polieder \mathcal{P} . V nekonveksnem primeru je Eo_R lahko poljubno velik ali majhen.

Ključne besede: Polieder, rotacijska orbita, Eulerjeva karakteristika.

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