

Also available at <http://amc-journal.eu>
ISSN 1855-3966 (printed ed.) ISSN 1855-3974 (electronic edn.)
ARS MATHEMATICA CONTEMPORANEA 12 (2017) 301–314

Search for the end of a path in the d -dimensional grid and in other graphs

Dániel Gerbner*

*Hungarian Academy of Sciences, Alfréd Rényi Institute of Mathematics,
P.O.B. 127, Budapest H-1364, Hungary*

Balázs Keszegh

*Hungarian Academy of Sciences, Alfréd Rényi Institute of Mathematics,
P.O.B. 127, Budapest H-1364, Hungary* †

Dömötör Pálvölgyi

*Department of Pure Mathematics and Mathematical Statistics,
University of Cambridge, UK* ‡

Günter Rote

*Freie Universität Berlin, Institut für Informatik,
Takustraße 9, 14195 Berlin, Germany* §

Gábor Wiener

*Department of Computer Science and Information Theory, Budapest University of Technology and
Economics, Műegyetem rkp. 3., H-1111, Budapest, Hungary* ¶

*corresponding author

Supported by the National Research, Development and Innovation Office – NKFIH, grant no. PD 109537 and by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences *E-mail addresses:* gerbner.daniel@renyi.mta.hu (Dániel Gerbner), keszegh.balazs@renyi.mta.hu (Balázs Keszegh), dom@cs.elte.hu (Dömötör Pálvölgyi), rote@inf.fu-berlin.de (Günter Rote), wiener@cs.bme.hu (Gábor Wiener)

†Supported by the National Research, Development and Innovation Office – NKFIH, grant no. PD 108406, NN 102029 (EUROGIGA project GraDR 10-EuroGIGA-OP-003), NK 78439, K 116769, by the János Bolyai Research Scholarship of the Hungarian Academy of Sciences, and DAAD.

‡Supported by the National Research, Development and Innovation Office – NKFIH, grant no. PD 104386 and NN 102029 (EUROGIGA project GraDR 10-EuroGIGA-OP-003), the János Bolyai Research Scholarship of the Hungarian Academy of Sciences, and by the Marie Skłodowska-Curie action of the EU, under grant IF 660400.

§Supported by the ESF EUROCORES programme EuroGIGA-VORONOI, Deutsche Forschungsgemeinschaft (DFG): RO 2338/5-1.

¶Supported by the National Research, Development and Innovation Office – NKFIH, grant no. 108947, and the János Bolyai Research Scholarship of the Hungarian Academy of Sciences

Abstract: We consider the worst-case query complexity of some variants of certain PPAD-complete search problems. Suppose we are given a graph G and a vertex $s \in V(G)$. We denote the directed graph obtained from G by directing all edges in both directions by G' . D is a directed subgraph of G' which is unknown to us, except that it consists of vertex-disjoint directed paths and cycles and one of the paths originates in s . Our goal is to find an endvertex of a path by using as few queries as possible. A query specifies a vertex $v \in V(G)$, and the answer is the set of the edges of D incident to v , together with their directions.

We also show lower bounds for the special case when D consists of a single path. Our proofs use the theory of graph separators. Finally, we consider the case when the graph G is a grid graph. In this case, using the connection with separators, we give asymptotically tight bounds as a function of the size of the grid, if the dimension of the grid is considered as fixed. In order to do this, we prove a separator theorem about grid graphs, which is interesting on its own right.

Keywords: Separator, graph, search, grid

Math. Subj. Class.: 90B40, 05C85

Dostopno tudi na <http://amc-journal.eu>
ISSN 1855-3966 (tiskana izd.) ISSN 1855-3974 (elektronska izd.)
ARS MATHEMATICA CONTEMPORANEA 12 (2017) 301–314

Iskanje konca poti v d -dimenzionalni rešetki in v drugih grafih

Dániel Gerbner*

*Hungarian Academy of Sciences, Alfréd Rényi Institute of Mathematics,
P.O.B. 127, Budapest H-1364, Hungary*

Balázs Keszegh

*Hungarian Academy of Sciences, Alfréd Rényi Institute of Mathematics,
P.O.B. 127, Budapest H-1364, Hungary* †

Dömötör Pálvölgyi

*Department of Pure Mathematics and Mathematical Statistics,
University of Cambridge, UK* ‡

Günter Rote

*Freie Universität Berlin, Institut für Informatik,
Takustraße 9, 14195 Berlin, Germany* §

Gábor Wiener

*Department of Computer Science and Information Theory, Budapest University of Technology and
Economics, Műegyetem rkp. 3., H-1111, Budapest, Hungary* ¶

*kontaktnei avtor

Podprto s strani National Research, Development and Innovation Office – NKFIH, sredstva št. PD 109537 in s strani János Bolyai Research Scholarship of the Hungarian Academy of Sciences *E-mail addresses:* gerbner.daniel@renyi.mta.hu (Dániel Gerbner), keszegh.balazs@renyi.mta.hu (Balázs Keszegh), dom@cs.elte.hu (Dömötör Pálvölgyi), rote@inf.fu-berlin.de (Günter Rote), wiener@cs.bme.hu (Gábor Wiener)

†Podprto s strani National Research, Development and Innovation Office – NKFIH, sredstva št. PD 108406, NN 102029 (EUROGIGA project GraDR 10-EuroGIGA-OP-003), NK 78439, K 116769, s strani János Bolyai Research Scholarship of the Hungarian Academy of Sciences, in DAAD.

‡Podprto s strani National Research, Development and Innovation Office – NKFIH, sredstva št. PD 104386 in NN 102029 (EUROGIGA project GraDR 10-EuroGIGA-OP-003), the János Bolyai Research Scholarship of the Hungarian Academy of Sciences, ter s strani Marie Skłodowska-Curie action of the EU, iz sredstev IF 660400.

§Podprto s strani ESF EUROCORES programme EuroGIGA-VORONOI, Deutsche Forschungsgemeinschaft (DFG): RO 2338/5-1.

¶Podprto s strani National Research, Development and Innovation Office – NKFIH, sredstva št. 108947, in s strani János Bolyai Research Scholarship of the Hungarian Academy of Sciences

Povzetek: Obravnavamo najslabši primer iskalne kompleksnosti nekaterih različic določenih PPAD-polnih iskalnih problemov. Denimo da je dan graf G in vozlišče $s \in V(G)$. Označimo usmerjeni graf dobljen iz grafa G z usmeritvijo vseh povezav v obe smeri z G' . D je usmerjen podgraf grafa G' , ki nam je neznan, vemo le, da sestoji iz vozliščno-disjunktnih usmerjenih poti in ciklov, in da se ena od poti začne v s . Naš cilj je najti končno vozlišče poti, za to pa uporabiti kar se malo iskanj. Iskanje specificira vozlišče $v \in V(G)$, in odgovor je množica povezav grafa D incidentnih vozlišču v , skupaj z njihovimi smermi.

Določimo tudi spodnje meje za posebni primer, ko D sestoji iz ene same poti. Naši dokazi uporabljajo teorijo grafovskih ločevalcev. Nazadnje obravnavamo primer, ko je graf G rešetkast graf. V tem primeru, uporabljajoč zvezo z ločevalci, podamo asimptotsko tesne meje, izražene z velikostjo rešetke, če je dimenzija rešetke smatrana za fiksno. V ta namen dokažemo ločevalske izreke v zvezi z rešetkastimi grafi, ki so zanimivi tudi sami po sebi.

Ključne besede: Ločevalec, graf, iskanje, rešetka.

Math. Subj. Class.: 90B40, 05C85