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## Independent sets on the Towers of Hanoi graphs\*

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**Abstract:** The number of independent sets is equivalent to the partition function of the hard-core lattice gas model with nearest-neighbor exclusion and unit activity. In this article, we mainly study the number of independent sets  $i(H_n)$  on the Tower of Hanoi graph  $H_n$  at stage  $n$ , and derive the recursion relations for the numbers of independent sets. Upper and lower bounds for the asymptotic growth constant  $\mu$  on the Towers of Hanoi graphs are derived in terms of the numbers at a certain stage, where  $\mu = \lim_{v \rightarrow \infty} \frac{\ln i(G)}{v(G)}$  and  $v(G)$  is the number of vertices in a graph  $G$ . Furthermore, we also consider the number of independent sets on the Sierpiński graphs which contain the Towers of Hanoi graphs as a special case.

**Keywords:** Independent sets, the Tower of Hanoi graph, Sierpiński graph, recursion relation, asymptotic growth constant, asymptotic enumeration.

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## Neodvisne množice na grafih Hanojskih stolpov<sup>‡</sup>

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**Povzetek:** Število neodvisnih množic je ekvivalentno particijski funkciji trdne jedrne mreže plinskega modela z izključitvijo najbližjega sosedu in enotsko aktivnostjo. V tem članku v glavnem študiramo število neodvisnih množic  $i(H_n)$  grafa Hanojskih stolpov  $H_n$  stopnje  $n$  in izpeljemo rekurzivne zveze za število neodvisnih množic. Izpeljemo zgornje in spodnje meje za konstanto  $\mu$  asimptotske rasti na grafih Hanojskih stolpov, izražene s števili na določeni stopnji, kjer je  $\mu = \lim_{v \rightarrow \infty} \frac{\ln i(G)}{v(G)}$  in je  $v(G)$  število vozlišč v grafu  $G$ . Obravnavamo tudi število neodvisnih množic na grafih Sierpińskega, ki vsebujejo grafe Hanojskih stolpov kot poseben primer.

**Ključne besede:** Neodvisne množice, grafi Hanojskih stolpov, graf Sierpińskega, rekurzivna zveza, konstanta asimptotske rasti, asimptotsko preštevanje.

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