Independent sets on the Towers of Hanoi graphs[*]

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Abstract: The number of independent sets is equivalent to the partition function of the hard-core lattice gas model with nearest-neighbor exclusion and unit activity. In this article, we mainly study the number of independent sets $i(H_n)$ on the Tower of Hanoi graph H_n at stage n, and derive the recursion relations for the numbers of independent sets. Upper and lower bounds for the asymptotic growth constant μ on the Towers of Hanoi graphs are derived in terms of the numbers at a certain stage, where $\mu = \lim_{v \to \infty} \frac{\ln i(G)}{v(G)}$ and $v(G)$ is the number of vertices in a graph G. Furthermore, we also consider the number of independent sets on the Sierpiński graphs which contain the Towers of Hanoi graphs as a special case.

Keywords: Independent sets, the Tower of Hanoi graph, Sierpiński graph, recursion relation, asymptotic growth constant, asymptotic enumeration.

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Neodvisne množice na grafih Hanojskih stolpov[‡]

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Povzetek: Število neodvisnih množic je ekvivalentno particijski funkciji trdne jedrne mreže plinskega modela z izključitvijo najbližjega soseda in enotsko aktivnostjo. V tem članku v glavnem študiramo število neodvisnih množic $i(H_n)$ grafa Hanojskih stolpov H_n stopnje n in izpeljemo rekurzivne zveze za število neodvisnih množic. Izpeljemo zgornje in spodnje meje za konstanto μ asimptotske rasti na grafih Hanojskih stolpov, izražene s števili na določeni stopnji, kjer je $\mu = \lim_{v \to \infty} \frac{\ln i(G)}{v(G)}$ in je $v(G)$ število vozlišč v grafu G. Obravnavamo tudi število neodvisnih množic na grafih Sierpińskega, ki vsebujejo grafe Hanojskih stolpov kot poseben primer.

Ključne besede: Neodvisne množice, grafi Hanojskih stolpov, graf Sierpińskega, rekurzivna zveza, konstanta asimptotske rasti, asimptotsko preštevanje.

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