

On convergence of binomial means, and an application to finite Markov chains

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Abstract: For a sequence $\{a_n\}_{n \geq 0}$ of real numbers, we define the sequence of its arithmetic means $\{a_n^*\}_{n \geq 0}$ as the sequence of averages of the first n elements of $\{a_n\}_{n \geq 0}$. For a parameter $0 < p < 1$, we define the sequence of p -binomial means $\{a_n^p\}_{n \geq 0}$ of the sequence $\{a_n\}_{n \geq 0}$ as the sequence of p -binomially weighted averages of the first n elements of $\{a_n\}_{n \geq 0}$. We compare the convergence of sequences $\{a_n\}_{n \geq 0}$, $\{a_n^*\}_{n \geq 0}$ and $\{a_n^p\}_{n \geq 0}$ for various $0 < p < 1$, i.e., we analyze when the convergence of one sequence implies the convergence of the other.

While the sequence $\{a_n^*\}_{n \geq 0}$, known also as the sequence of Cesàro means of a sequence, is well studied in the literature, the results about $\{a_n^p\}_{n \geq 0}$ are hard to find. Our main result shows that, if $\{a_n\}_{n \geq 0}$ is a sequence of non-negative real numbers such that $\{a_n^p\}_{n \geq 0}$ converges to $a \in \mathbb{R} \cup \{\infty\}$ for some $0 < p < 1$, then $\{a_n^*\}_{n \geq 0}$ also converges to a . We give an application of this result to finite Markov chains.

Keywords: Sequence, convergence, Cesàro mean, binomial mean, finite Markov chain.

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O konvergenci binomskih sredin in aplikacija na končne Markovske verige

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Povzetek: Za zaporedje $\{a_n\}_{n \geq 0}$ realnih števil definiramo zaporedje njegovih aritmetičnih sredin $\{a_n^*\}_{n \geq 0}$ kot zaporedje povprečij prvih n členov zaporedja $\{a_n\}_{n \geq 0}$. Za parameter $0 < p < 1$ definiramo zaporedje p -binomskih sredin $\{a_n^p\}_{n \geq 0}$ zaporedja $\{a_n\}_{n \geq 0}$ kot zaporedje p -binomsko uteženih povprečij prvih n členov zaporedja $\{a_n\}_{n \geq 0}$. Primerjamo konvergenco zaporedij $\{a_n\}_{n \geq 0}$, $\{a_n^*\}_{n \geq 0}$ in $\{a_n^p\}_{n \geq 0}$ za različne $0 < p < 1$ oziroma analiziramo, kdaj konvergenca enega zaporedja implicira konvergenco drugega.

Medtem ko je zaporedje $\{a_n^*\}_{n \geq 0}$, znano tudi kot zaporedje Cesàrovih sredin zaporedja, dobro raziskano v literaturi, pa je rezultate v zvezi z $\{a_n^p\}_{n \geq 0}$ težko najti. Naš glavni rezultat pokaže, da če je $\{a_n\}_{n \geq 0}$ zaporedje nenegativnih realnih števil in $\{a_n^p\}_{n \geq 0}$ konvergira k $a \in \mathbb{R} \cup \{\infty\}$ za neki $0 < p < 1$, potem $\{a_n^*\}_{n \geq 0}$ prav tako konvergira k a . Podamo aplikacijo tega rezultata v končnih Markovskih verigah.

Ključne besede: Zaporedje, konvergenca, Cesàrova sredina, binomska sredina, končna Markovska veriga.

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