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On the split structure of lifted groups

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Abstract: Let p: $\tilde{X} \to X$ be a regular covering projection of connected graphs with the group of covering transformations CT_p being abelian. Assuming that a group of automorphisms $G \leq \mathrm{Aut}(X)$ lifts along p to a group $\tilde{G} \leq \mathrm{Aut}(\tilde{X})$, the problem whether the corresponding exact sequence id $\to CT_p \to \tilde{G} \to G \to$ id splits is analyzed in detail in terms of a Cayley voltage assignment that reconstructs the projection up to equivalence.

In the above combinatorial setting the extension is given only implicitly: neither $\tilde{G}$ nor the action $G \to \mathrm{Aut}\, CT_p$ nor a 2-cocycle $G \times G \to CT_p$, are given. Explicitly constructing the cover $\tilde{X}$ together with CT_p and $\tilde{G}$ as permutation groups on $\tilde{X}$ is time and space consuming whenever CT_p is large; thus, using the implemented algorithms (for instance, HasComplement in Magma) is far from optimal. Instead, we show that the minimal required information about the action and the 2-cocycle can be effectively decoded directly from voltages (without explicitly constructing the cover and the lifted group); one could then use the standard method by reducing the problem to solving a linear system of equations over the integers. However, along these lines we here take a slightly different approach which even does not require any knowledge of cohomology. Time and space complexity are formally analyzed whenever CT_p is elementary abelian.

Keywords: Algorithm, abelian cover, Cayley voltages, covering projection, graph, group extension, group presentation, lifting automorphisms, linear systems over the integers, semidirect product.

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O razcepni strukturi dvignjenih grup

Povzetek: Naj bo p: $\tilde{X}$ → X regularna krovna projekcija povezanih grafov, grupa krovnih transformacij CT_p pa naj bo abelova. Ob predpostavki, da se grupa avtomorfizmov G <= Aut (X) dvigne vzdolž p do grupe $\tilde{G}$ <= Aut($\tilde{X}$), podrobno analiziramo problem, ali se ustrezno eksaktno zaporedje id → CT_p → $\tilde{G}$ → G → id razcepi glede na Cayleyevo dodelitev napetosti, ki rekonstruira projekcijo do ekvivalence natančno.

V gornjem kombinatoričnem sestavu je razširitev podana samo implicitno: podani niso ne $\tilde{G}$ ne delovanje G → Aut CT_p ne 2-kocikel G x G → CT_p. Eksplicitno konstruiranje krova $\tilde{X}$ ter CT_p in $\tilde{G}$ kot permutacijskih grup na $\tilde{X}$ je časovno in prostorsko zahtevno vselej, kadar je CT_p velik; tako je uporaba implementiranih algoritmov (na primer, HasComplement v Magmi) vse prej kot optimalna. Namesto tega pokažemo, da lahko najnujnejšo informacijo o delovanju in 2-kociklu učinkovito izluščimo neposredno iz napetosti (ne da bi eksplicitno konstruirali krov in dvignjeno grupo); zdaj bi bilo mogoče uporabiti standardno metodo reduciranja problema na reševanje sistema linearnih enačb nad celimi števili. Vendar tukaj uberemo malce drugačen pristop, ki sploh ne zahteva nobenega poznavanja kohomologije. Časovno in prostorsko zahtevnost formalno analiziramo za vse primere, ko je CT_p elementarna abelova.

Ključne besede: Algoritem, abelov krov, Cayleyeve napetosti, krovna projekcija, graf, razširitev grupe, prezentacija grupe, dvig avtomorfizmov, sistemi linearnih enačb nad celimi števili, poldirekten produkt.