

Also available at <http://amc-journal.eu>
ISSN 1855-3966 (printed edn.), ISSN 1855-3974 (electronic edn.)
ARS MATHEMATICA CONTEMPORANEA 8 (2015) 215–223

On automorphism groups of graph truncations

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Abstract: It is well known that the Petersen graph, the Coxeter graph, as well as the graphs obtained from these two graphs by replacing each vertex with a triangle, are trivalent vertex-transitive graphs without Hamilton cycles, and are indeed the only known connected vertex-transitive graphs of valency at least two without Hamilton cycles. It is known by many that the replacement of a vertex with a triangle in a trivalent vertex-transitive graph results in a vertex-transitive graph if and only if the original graph is also arc-transitive. In this paper, we generalize this notion to t -regular graphs Γ and replace each vertex with a complete graph K_t on t vertices. We determine necessary and sufficient conditions for $T(\Gamma)$ to be hamiltonian, show $\text{Aut}(T(\Gamma)) \cong \text{Aut}(\Gamma)$, as well as show that if Γ is vertex-transitive, then $T(\Gamma)$ is vertex-transitive if and only if Γ is arc-transitive. Finally, in the case where t is prime we determine necessary and sufficient conditions for $T(\Gamma)$ to be isomorphic to a Cayley graph as well as an additional necessary and sufficient condition for $T(\Gamma)$ to be vertex-transitive.

Keywords: Truncation, automorphism group, Cayley graph, Hamiltonian.

O grupi avtomorfizmov prisekanj grafa

Povzetek: Znano je, da so Petersenov graf, Coxeterjev graf, pa tudi grafi, dobljeni iz njiju z nadomestitvijo vsakega vozlišča s trikotnikom, trivalentni vozliščno-tranzitivni grafi brez Hamiltonovih ciklov; to so tudi edini znani povezani vozliščno-tranzitivni grafi valence najmanj dve brez Hamiltonovih ciklov. Znano je tudi, da zamenjava vozlišča s trikotnikom v trivalentnem vozliščno-tranzitivnem grafu da vozliščno-tranzitivni graf če in samo če je prvotni graf tudi ločno-tranzitivni. V tem članku posplošimo ta koncept na t -regularne grafe Γ in nadomestimo vsako vozlišče s polnim grafom K_t na t vozliščih. Določimo potrebne in zadostne pogoje za to, da je $T(\Gamma)$ hamiltonski, pokažemo, da je $\text{Aut}(T(\Gamma)) \cong \text{Aut}(\Gamma)$, pokažemo pa tudi, da če je Γ vozliščno-tranzitivni, potem je $T(\Gamma)$ vozliščno-tranzitivni če in samo če je Γ ločno-tranzitivni. V primeru, ko je t praštevilo, določimo potrebne in zadostne pogoje za to, da je $T(\Gamma)$ izomorfen Cayleyevemu grafu, pa tudi dodaten potreben in zadosten pogoj za to, da je $T(\Gamma)$ vozliščno-tranzitivni.

Ključne besede: Prisekanje, grupa avtomorfizmov, Cayleyev graf, Hamiltonov graf.

Math. Subj. Class.: 05C25