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## Classification of the regular oriented hypermaps with prime number of hyperfaces

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**Abstract:** Regular oriented hypermaps are triples  $(G; a, b)$  consisting of a finite 2-generated group  $G$  and a pair  $a, b$  of generators of  $G$ , where the left cosets of  $\langle a \rangle$ ,  $\langle b \rangle$  and  $\langle ab \rangle$  describe respectively the hyperfaces, hypervertices and hyperedges. They generalise regular oriented maps (triples with  $ab$  of order 2) and describe cellular embeddings of regular hypergraphs on orientable surfaces. Previously, we have classified the regular oriented hypermaps with a prime number of hyperfaces and with no non-trivial regular proper quotients with the same number of hyperfaces (i.e. primer hypermaps with prime number of hyperfaces), which generalises the classification of regular oriented maps with prime number of faces and underlying simple graph. Now we classify the regular oriented hypermaps with a prime number of hyperfaces. As a result of this classification, we conclude that the regular oriented hypermaps with prime  $p$  hyperfaces have metacyclic automorphism groups and the chiral ones have cyclic chirality

groups; of these the "canonical metacyclic" (i.e. those for which  $\langle a \rangle$  is normal in  $G$ ) have chirality index a divisor of  $n$  (the hyperface valency) and the non "canonical metacyclic" have chirality index  $p$ . We end the paper by counting, for each positive integer  $n$  and each prime  $p$ , the number of regular oriented hypermaps with  $p$  hyperfaces of valency  $n$ .

**Keywords:** Hypermaps, maps, hypergraphs, regularity, orientably regular, chirality.

Math. Subj. Class.: 05E18, 05E15, 20B25, 05C25, 05C30

## Klasifikacija regularnih orientiranih hiperzemljevidov s praštevilskim številom hiperlic

**Povzetek:** Regularni orientirani hiperzemljevdi so trojice  $(G; a, b)$ , ki sestojijo iz končne 2-generirane grupe  $G$  in para  $a, b$  generatorjev grupe  $G$ , pri čemer levi odseki  $\langle a \rangle$ ,  $\langle b \rangle$ , in  $\langle ab \rangle$  opisujejo po vrsti hiperlica, hipervozlišča in hiperpovezave. Predstavljajo posplošitev regularnih orientiranih zemljevdi (trojic, pri katerih ima  $ab$  red 2) in opisujejo celične vložitve regularnih hipergrafov v orientabilne ploskve. Predhodno smo že klasificirali regularne orientirane hiperzemljevde s praštevilskim številom hiperlic in brez netrivialnih regularnih pravih kvocientov z istim številom hiperlic (tj. praštevilskih hiperzemljevdi s praštevilskim številom hiperlic), kar posplošuje klasifikacijo regularnih orientiranih zemljevdi s praštevilskim številom lic in enostavnim osnovnim grafom. Tukaj klasificiramo regularne orientirane hiperzemljevde s praštevilskim številom hiperlic. Iz te klasifikacije izhaja, da imajo regularni orientirani hiperzemljevdi s praštevilskim številom  $p$  hiperlic metaciklične grupe avtomorfizmov, kiralni pa imajo ciklične kiralne grupe; med temi imajo "kanonično metaciklične" (tj. tiste, pri katerih je  $\langle a \rangle$  edinka v  $G$ ) kiralnostni indeks tak, da deli  $n$  (valenco hiperlic), tiste pa, ki niso "kanonično metaciklične" imajo kiralnostni indeks  $p$ . Članek zaključimo tako, da določimo število regularnih orientiranih hiperzemljevdi s  $p$  hiperlici valence  $n$ , kjer je  $n$  poljubno naravno število in  $p$  praštevilo.

**Ključne besede:** Hiperzemljevdi, zemljevdi, hipergrafi, regularnost, orientabilno regularen, kiralnost.

