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Reachability relations, transitive digraphs and groups

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Abstract

In [6] it was shown that properties of digraphs such as growth, property Z, and number of ends are reflected by the properties of certain reachability relations defined on the vertices of the corresponding digraphs.

In this paper we study these relations in connection with certain properties of automorphism groups of transitive digraphs. In particular, one of the main results shows that if a transitive digraph admits a nilpotent subgroup of automorphisms with finitely many orbits, then its nilpotency class and the number of orbits are closely related to particular properties of reachability relations defined on the digraphs in question.

The obtained results have interesting implications for Cayley digraphs of certain types of groups such as torsion-free groups of polynomial growth.

Keywords: Cayley digraph, reachability relation.

Math. Subj. Class.: 05C20, 05C25

Dosegljivostne relacije, tranzitivni digrafi in grupe

Povzetek: V [6] je bilo pokazano, da se lastnosti usmerjenih grafov, kot so rast, lastnost Z, in število koncev odražajo v lastnostih določenih dosegljivostnih relacij definiranih na vozliščih ustreznih digrafov.

V tem članku obravnavamo te relacije v povezavi z določenimi lastnostmi grup avtomorfizmov tranzitivnih digrafov. Tako npr. eden od glavnih rezultatov pove, da če tranzitivni digraf dopušča nilpotentno podgrupo avtomorfizmov s končno mnogo orbitami, potem sta njegov nilpotentni razred in število orbit tesno povezana z določenimi lastnostmi dosegljivostnih relacij, definiranih na ustreznih digrafih.

Dobljeni rezultati imajo zanimive implikacije za Cayleyeve digrafe določenih tipov grup, kot so npr. grupe brez torzije s polinomske rastjo.

Ključne besede: Cayleyev digraf, dosegljivostna relacija.