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Rational sums of hermitian squares of free noncommutative polynomials

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Abstract: In this paper we consider polynomials in noncommuting variables that admit sum of hermitian squares and commutators decompositions. We recall algorithms for finding decompositions of this type that are based on semidefinite programming. The main part of the article investigates how to find such decomposition with *rational coefficients* if the original polynomial has rational coefficients. We show that the numerical evidence, obtained by the Gram matrix method and semidefinite programming, which is usually an almost feasible point, can be frequently tweaked to obtain an *exact certificate* using rational numbers. In the presence of Slater points, the Peyrl-Parrilo rounding and projecting method applies. On the other hand, in the absence of strict feasibility, a variant of the facial reduction is proposed to reduce the size of the semidefinite program and to enforce the existence of Slater points. All these methods are implemented in our open source computer algebra package NCSOSTools. Throughout the paper many worked out examples are presented to illustrate our results.

Keywords: Sum of squares, semidefinite programming, noncommutative polynomial, Matlab toolbox, commutator, cyclic equivalence, free positivity, real algebraic geometry, Motzkin polynomial, Bessis- Moussa-Villani (BMV) conjecture, NCSOSTools.

Math. Subj. Class.: 13J30, 90C22; 08B20, 11E25, 90C90

Racionalne vsote hermitskih kvadratov prostih nekomutativnih polinomov

Povzetek: V tem članku obravnavamo polinome v nekomutirajočih spremenljivkah, ki dovoljujejo vsoto hermitskih kvadratov in komutatorske dekompozicije. Spomnimo na algoritme za iskanje dekompozicij tega tipa, osnovanih na semidefinitnem programiranju. Glavni del članka preučuje, kako najti takšne dekompozicije z *racionalnimi koeficienti*, če ima tudi prvotni polinom racionalne koeficiente. Pokažemo, da lahko s pomočjo numeričnih kazalcev, dobljenih z uporabo Gramove matrike in semidefinitnim programiranjem, kar je ponavadi bolj ali manj izvedljivo, pogosto pridobimo *eksakten certifikat* z uporabo racionalnih števil. Pri Slaterjevih točkah se da uporabiti Peyrl-Parrilovo zaokrožanje in projektne metode. Po drugi strani, kadar nimamo stroge izvedljivosti, sibility, predlagamo varianto redukcije lic, s katero reduciramo velikost semidefinitnega programa in vsilimo eksistenco Slaterjevih točk. Vse te metode so implementirane na našem odprtokodnem paketu računske algebre NCSOSTools. V samem članku predstavimo številne izdelane primere, ki ponazarjajo naše rezultate.

Ključne besede: Vsota kvadratov, semidefinitno programiranje, nekomutativni polinom, Matlab orodje, komutator, ciklična ekvivalenca, prosta pozitivnost, realna algebraična geometrija, Motzkinov polinom, Bessis- Moussa-Villanijeva (BMV) domneva, NCSOStools.