

Also available at <http://amc-journal.eu>
ISSN 1855-3966 (printed edn.), ISSN 1855-3974 (electronic edn.)
Ars Mathematica Contemporanea Volume 1, Issue 2, Year 2008, Pages 169-194

Relating Embedding and Coloring Properties of Snarks

Bojan Mohar, Eckhard Steffen, Andrej Vodopivec

Abstract

In 1969, Grünbaum conjectured that snarks do not have polyhedral embeddings into orientable surfaces. To describe the deviation from polyhedrality, we define the defect of a graph and use it to study embeddings of superpositions of cubic graphs into orientable surfaces. Superposition was introduced in [4] to construct snarks with arbitrary large girth. It is shown that snarks constructed in [4] do not have polyhedral embeddings into orientable surfaces. For each $k \geq 2$ we construct infinitely many snarks with defect precisely k . We then relate the defect with the resistance $r(G)$ of a cubic graph G which is the size of a minimum color class of a 4-edge-coloring of G . These results are then extended to deal with some weaker versions of the Grünbaum Conjecture.

Keywords: Graph embedding, resistance, snark, superposition.

Math. Subj. Class.: 05C10, 05C15

Math Sci Net: [05C10 \(05C15\)](#)

Primerjava vložljivostnih in barvnih lastnosti snarkov

Povzetek

Leta 1969 je Grünbaum postavil domnevo, da snarki nimajo poliedrskih vložitev v orientabilne ploskve. Za opis odklona od poliedrskosti definiramo defekt grafa in ga uporabimo za študij vložitev superpozicij kubičnih grafov v orientabilne ploskve. Superpozicijo so vpeljali v [4] za konstruiranje snarkov z poljubno velikim obsegom. Pokažemo, da snarki, konstruirani v [4], nimajo poliedrskih vložitev v orientabilne ploskve. Za vsak $k \geq 2$ konstruiramo neskončno mnogo snarkov z defektom natanko k . Nato povežemo defekt z rezistenco $r(G)$ kubičnega grafa G , ki pomeni velikost najmanjšega barvnega razreda 4-barvanja povezav grafa G . Te rezultate potem razširimo in obravnavamo nekaj šibkejših različic Grünbaumove domneve.

Ključne besede: Vložitev grafa, rezistenca, snark, superpozicija.