

Comparing Wiener, Szeged and revised Szeged index on cactus graphs

Stefan Hammer* 

Graz University of Technology, Rechbauerstraße 12, Graz, Austria

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Abstract

We show that on cactus graphs the Szeged index is bounded above by twice the Wiener index. For the revised Szeged index the situation is reversed if the graph class is further restricted. Namely, if all blocks of a cactus graph are cycles, then its revised Szeged index is bounded below by twice its Wiener index. Additionally, we show that these bounds are sharp and examine the cases of equality. Along the way, we provide a formulation of the revised Szeged index as a sum over vertices, which proves very helpful, and may be interesting in other contexts.

Keywords: Wiener index, (Revised) Szeged index, cactus graphs.

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E-mail address: stefan.hammer@tugraz.at (Stefan Hammer)

Primerjava Wienerjevega, Szegedovega in revidiranega Szegedovega indeksa na kaktusnih grafih

Stefan Hammer* 

Graz University of Technology, Rechbauerstraße 12, Graz, Austria

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Povzetek

Pokažemo, da je na kaktusnih grafih Szegedov indeks omejen navzgor z dvakratnikom Wienerjevega indeksa. Za revidirani Szegedov indeks je situacija obrnjena, če razred grafov še dodatno omejimo. Velja namreč naslednje: če so vsi bloki kaktusnega grafa cikli, potem je njegov revidirani Szegedov indeks omejen navzdol z dvakratnikom Wienerjevega indeksa. Dodatno pokažemo, da so te meje ostre, in raziščemo primere, v katerih dejansko velja enakost. Spotoma podamo formulacijo revidiranega Szegedovega indeksa v smislu vsote nad točkami, kar se izkaže za zelo koristno, in bi lahko bilo zanimivo v drugih kontekstih.

Ključne besede: Wienerjev indeks, (revidirani) Szegedov indeks, kaktusni grafi.

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E-poštni naslov: stefan.hammer@tugraz.at (Stefan Hammer)