

The search for small association schemes with noncyclotomic eigenvalues*

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Received 9 November 2021, accepted 4 November 2022, published online 11 January 2023

Abstract

In this article we determine feasible parameter sets for (what could potentially be) commutative association schemes with noncyclotomic eigenvalues that are of smallest possible rank and order. A feasible parameter set for a commutative association scheme corresponds to a standard integral table algebra with integral multiplicities that satisfies all of the parameter restrictions known to hold for association schemes. For each rank and involution type, we generate an algebraic set for which any suitable integral solution corresponds to a standard integral table algebra with integral multiplicities, and then try to find the smallest suitable solution. The main results of this paper show the eigenvalues of association schemes of rank 4 and nonsymmetric association schemes of rank 5 will always be cyclotomic. In the rank 5 cases, the results rely on calculations done by computer for Gröbner bases or for bases of rational vector spaces spanned by polynomials. We give several examples of feasible parameter sets for small symmetric association schemes of rank 5 that have noncyclotomic eigenvalues.

Keywords: Association schemes, table algebras, character tables.

Math. Subj. Class. (2020): 05E30, 13P15

*We would like to thank anonymous referees for carefully reading the manuscript and for their insightful comments

[†]This author's work was supported by an NSERC Discovery Grant.

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Iskanje majhnih asociativnih shem z neciklotomskimi lastnimi vrednostmi*

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Prejeto 9. novembra 2021, sprejeto 4. novembra 2022, objavljeno na spletu 11. januarja 2023

Povzetek

V tem članku določimo dopustne množice parametrov za (to, kar bi potencialno lahko bile) tiste komutativne asociativne sheme z neciklotomskimi lastnimi vrednostmi, ki imajo najmanjši možni rang in red. Dopustna množica parametrov za komutativne asociativne sheme odgovarja standardni celoštevilski tabelni algebri s celoštevilskimi večkratnostmi, ki zadošča vsem parametrskim omejitvam, za katere se ve, da veljajo za asociativne sheme. Za vsak tip ranga in involucije generiramo algebraično množico, katere poljubna primerna celoštevilska rešitev odgovarja standardni celoštevilski tabelni algebri s celoštevilskimi večkratnostmi, potem pa poskušamo poiskati najmanjšo možno rešitev. Glavni rezultati tega članka pokažejo, da so lastne vrednosti asociativnih shem ranga 4 in nesimetričnih asociativnih shem ranga 5 vselej ciklotomske. V primerih ranga 5 rezultati temeljijo na računalniških izračunih za Gröbnerjeve baze ali za baze racionalnih vektorskih prostorov, razpetih s polinomi. Predstavimo več primerov dopustnih množic parametrov za majhne simetrične asociativne sheme ranga 5 z neciklotomskimi lastnimi vrednostmi.

Ključne besede: Asociativnih sheme, tabelne algebre, tabele karakterjev.

Math. Subj. Class. (2020): 05E30, 13P15

*Rada bi se zahvalila neznanim recenzentom za skrbno branje rokopisa in za njihove pronicljive pripombe.

[†] Delo tega avtorja je bilo podprto z raziskovalno dotacijo NSERC Discovery Grant.

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