

# Double generalized majorization and diagrammatics\*

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## Abstract

In this paper we show that the generalized majorization of partitions of integers has a surprising completing-squares property. Together with the previously obtained transitivity-like property, this enables a compelling diagrammatical interpretation. Apart from purely combinatorial interest, the main result has applications in matrix completion problems, and representation theory of quivers.

*Keywords:* Partitions, majorization, diagrammatics, inequalities.

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# Dvojna posplošena majorizacija in diagramatika\*

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## Povzetek

V tem članku dokažemo, da ima posplošena majorizacija particij presenetljivo lastnost dopolnitve kvadratov. Skupaj s predhodno dobljeno lastnostjo, podobno tranzitivnosti, to omogoča prepričljivo diagramsko interpretacijo. Poleg čisto kombinatoričnega prispevka je glavni rezultat uporaben pri problemih matrične dopolnitve ter v predstavitveni teoriji tulcev.

*Ključne besede: Particije, majorizacija, diagramatska predstavitev, neenakosti.*

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