

# Almost simple groups as flag-transitive automorphism groups of symmetric designs with $\lambda$ prime\*

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Received 19 August 2021, accepted 10 January 2023, published online 28 February 2023

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## Abstract

In this article, we study symmetric designs with  $\lambda$  prime admitting a flag-transitive and point-primitive automorphism group  $G$  of almost simple type with socle  $X$ . We prove that either  $\mathcal{D}$  is one of the six well-known examples of biplanes and triplanes, or  $\mathcal{D}$  is the point-hyperplane design of  $\text{PG}(n-1, q)$  with  $\lambda = (q^{n-2} - 1)/(q - 1)$  prime and  $X = \text{PSL}_n(q)$ .

*Keywords:* Almost simple group, automorphism group, flag-transitive, point-primitive, symmetric design.

*Math. Subj. Class. (2020):* 05B05, 05B25, 20B25, 20D05

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\*The authors would like to thank anonymous referees for providing us helpful and constructive comments and suggestions.

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# Skoraj enostavne grupe kot praporno tranzitivne grupe avtomorfizmov simetričnih dizajnov, kjer je $\lambda$ praštevilo\*

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Prejeto 19. avgusta 2021, sprejeto 10. januarja 2023, objavljeno na spletu 28. februarja 2023

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## Povzetek

V tem članku raziskujemo simetrične dizajne s praštevilskim parametrom  $\lambda$ , ki dopuščajo praporno tranzitivno točkovno primitivno grupo avtomorfizmov  $G$  skoraj enostavnega tipa s podstavkom  $X$ . Dokažemo, da velja naslednje: bodisi je  $\mathcal{D}$  eden od šestih dobro znanih primerov dvojnih ravnin ali trojnih ravnin bodisi je  $\mathcal{D}$  točkovno hiperravninski dizajn, pripadajoč  $PG(n-1, q)$ , kjer je  $\lambda = (q^{n-2} - 1)/(q - 1)$  praštevilo in  $X = PSL_n(q)$ .

*Ključne besede:* Skoraj enostavna grupa, grupa avtomorfizmov, praporno tranzitiven, točkovno primitiven, simetrični dizajn.

*Math. Subj. Class. (2020):* 05B05, 05B25, 20B25, 20D05

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\*Avtorji bi se radi zahvalili recenzentom za njihove koristne in konstruktivne pripombe in predloge.

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