

On the essential annihilating-ideal graph of commutative rings*

Mohd Nazim , Nadeem ur Rehman [†] 

Department of Mathematics, Aligarh Muslim University, Aligarh-202002, India

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Abstract

Let R be a commutative ring with unity, $A(R)$ be the set of annihilating-ideals of R and $A^*(R) = A(R) \setminus \{0\}$. In this paper, we introduced and studied the *essential annihilating-ideal graph* of R , denoted by $\mathcal{EG}(R)$, with vertex set $A^*(R)$ and two distinct vertices I_1 and I_2 are adjacent if and only if $\text{Ann}(I_1 I_2)$ is an essential ideal of R . We prove that $\mathcal{EG}(R)$ is a connected graph with diameter at most three and girth at most four if $\mathcal{EG}(R)$ contains a cycle. Furthermore, the rings R are characterized for which $\mathcal{EG}(R)$ is a star or a complete graph. Finally, we classify all the Artinian rings R for which $\mathcal{EG}(R)$ is isomorphic to some well-known graphs.

Keywords: Annihilating-ideal graph, zero-divisor graph, complete graph, planar graph, genus of a graph.

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[†]Corresponding author.

E-mail addresses: mnazim1882@gmail.com (Mohd Nazim), nu.rehman.mm@amu.ac.in (Nadeem ur Rehman)

O esencialnem grafu anihilirajočih idealov komutativnih kolobarjev*

Mohd Nazim , Nadeem ur Rehman [†] 

Department of Mathematics, Aligarh Muslim University, Aligarh-202002, India

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Povzetek

Naj bo R komutativen kolobar z enoto, $A(R)$ množica anihilirajočih idealov kolobarja R in $A^*(R) = A(R) \setminus \{0\}$. V tem članku vpeljemo in preučujemo *esencialni graf anihilirajočih idealov* kolobarja R , označen z $\mathcal{EG}(R)$; množica njegovih vozlišč je $A^*(R)$, dva različna vozlišča I_1 in I_2 tega grafa pa sta sosedna če in samo če je $Ann(I_1 I_2)$ esencialni ideal kolobarja R . Dokažemo, da je $\mathcal{EG}(R)$ povezan graf s premerom največ tri in ožino največ štiri, če $\mathcal{EG}(R)$ vsebuje cikel. Karakteriziramo kolobarje R , za katere je $\mathcal{EG}(R)$ zvezda ali pa polni graf. Klasificiramo tudi vse Artinske kolobarje R , za katere je $\mathcal{EG}(R)$ izomorfen nekaterim znanim grafom.

Ključne besede: Graf anihilirajočih idealov, graf deliteljev nič, polni graf, ravninski graf, rod grafa. Math. Subj. Class. (2020): 13A15, 05C10, 05C12, 05C25

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[†]Kontaktni avtor.

E-poštna naslova: mnazim1882@gmail.com (Mohd Nazim), nu.rehman.mm@amu.ac.in (Nadeem ur Rehman)