

The fullerene graphs with a perfect star packing*

Lingjuan Shi † 

*School of Software, Northwestern Polytechnical University,
Xi'an, Shaanxi 710072, P. R. China*

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Abstract

Fullerene graph G is a connected plane cubic graph with only pentagonal and hexagonal faces, which is the molecular graph of carbon fullerene. A spanning subgraph of G is called a perfect star packing in G if its each component is isomorphic to $K_{1,3}$. For an independent set $D \subseteq V(G)$, if each vertex in $V(G) \setminus D$ has exactly one neighbor in D , then D is called an efficient dominating set of G . In this paper we show that the number of vertices of a fullerene graph admitting a perfect star packing must be divisible by 8. This answers an open problem asked by Došlić et al. and also shows that a fullerene graph with an efficient dominating set has $8n$ vertices. In addition, we find some counterexamples for the necessity of Theorem 14 of paper of Došlić et al. from 2020 and list some subgraphs that preclude the existence of a perfect star packing of type P_0 .

Keywords: Fullerene graph, perfect star packing, efficient dominating set.


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E-mail address: shilj18@nwpu.edu.cn (Lingjuan Shi)

Fulerenski grafi s popolnim pakiranjem zvezd*

Lingjuan Shi † 

*School of Software, Northwestern Polytechnical University,
Xi'an, Shaanxi 710072, P. R. China*

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Povzetek

Fulerenski graf G je povezan ravninski kubični graf s samimi peterokotnimi in šestokotnimi lici, ki predstavlja molekularni graf ogljikovega fulerena. Vpeti podgraf grafa G se imenuje popolno zvezdno pakiranje v grafu G , če je vsaka njegova komponenta izomorfna $K_{1,3}$. Neodvisna množica $D \subseteq V(G)$, v kateri ima vsako vozlišče iz $V(G) \setminus D$ natanko enega sosedo v D , se imenuje učinkovita dominantna množica grafa G . V tem članku pokažemo, da mora biti število vozlišč fulerenskega grafa, ki dopušča popolno zvezdno pakiranje, deljivo z 8. To odgovarja na odprt problem, ki so ga zastavili Došlić in dr. in kaže, da ima fulerenski graf z učinkovito dominantno množico $8n$ vozlišč. Pokažemo tudi nekaj protiprimerov za nujnost izreka 14 v članku Došlić in dr. iz leta 2020 in prikažemo nekatere podgrafe, ki izključujejo obstoj popolnega zvezdnega pakiranja tipa $P0$.

Ključne besede: Fulerenski graf, popolno zvezdno pakiranje, učinkovita dominantna množica.

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E-poštni naslov: shilj18@nwpu.edu.cn (Lingjuan Shi)