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Poly-antimatroid polyhedra

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Abstract

The notion of “antimatroid with repetition” was conceived by Björner, Lovász and Shor in 1991 as an extension of the notion of antimatroid in the framework of non-simple languages. Further they were investigated by the name of “poly-antimatroids” (Nakamura, 2005, Kempner & Levit, 2007), where the set system approach was used. If the underlying set of a poly-antimatroid consists of n elements, then the poly-antimatroid may be represented as a subset of the integer lattice \mathbb{Z}^n . We concentrate on geometrical properties of two-dimensional ($n = 2$) poly-antimatroids - poly-antimatroid polygons, and prove that these polygons are parallelogram polyominoes. We also show that each two-dimensional poly-antimatroid is a poset poly-antimatroid, i.e., it is closed under intersection.

The convex dimension $cdim(S)$ of a poly-antimatroid S is the minimum number of maximal chains needed to realize S . While

the convex dimension of an n -dimensional poly-antimatroid may be arbitrarily large, we prove that the convex dimension of an n -dimensional poset poly-antimatroid is equal to n .

Keywords

antimatroid, polyhedron, convex dimension, lattice animal, polyomino

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Poli-antimatroidni poliedri

Povzetek

Pojem “antimatroida s ponavljanjem” so uvedli Bjorner, Lovasz in Shor leta 1991 kot razširitev pojma antimatroida v okviru ne-enostavnih jezikov. Nadalje so bili raziskovani pod imenom “poli-antimatroidi” (Nakamura, 2005, Kempner & Levit, 2007), kjer je bil uporabljen pristop sistema množic. Če osnovna množica poli-matroida sestoji iz n elementov, potem lahko poli-matroid predstavimo kot podmnožico celoštevilske mreže Z^n . Osredotočamo se na geometrijske lastnosti dvodimenzionalnih ($n = 2$) poli-antimatroidov oziroma na poli-antimatroidne poligone in dokažemo, da so ti poligoni paralelogramski poliomini. Pokažemo tudi, da je vsak dvo-

dimenzionalen poli-antimatroid delno urejen poli-antimatroid, kar pomeni, da je zaprt za preseke.

Konveksna dimenzija $cdim(S)$ poli-antimatroida S je minimalno število maksimalnih verig, potrebnih za realizacijo S . Medtem ko je konveksna dimenzija n -dimenzionalnega poli-antimatroida lahko poljubno velika, se izkaže, da je konveksna dimenzija n -dimenzionalnega delno urejenega poli-antimatroida enaka n .

Ključne besede

antimatroid, polieder, konveksna dimenzija, mrežna žival, poliomino