


Some remarks on the square graph of the hypercube

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Abstract

Let $\Gamma = (V, E)$ be a graph. The square graph Γ^2 of the graph Γ is the graph with the vertex set $V(\Gamma^2) = V$ in which two vertices are adjacent if and only if their distance in Γ is at most two. The square graph of the hypercube Q_n has some interesting properties. For instance, it is highly symmetric and panconnected. In this paper, we investigate some algebraic properties of the graph Q_n^2 . In particular, we show that the graph Q_n^2 is distance-transitive. We show that the graph Q_n^2 is an imprimitive distance-transitive graph if and only if n is an odd integer. Also, we determine the spectrum of the graph Q_n^2 . Finally, we show that when $n > 2$ is an even integer, then Q_n^2 is an automorphic graph, that is, Q_n^2 is a distance-transitive primitive graph which is not a complete or a line graph.


Keywords: Square of a graph, distance-transitive graph, hypercube, automorphism group, Johnson graph, automorphic graph.

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Nekaj pripomb v zvezi s kvadratnim grafom hiperkocke

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Povzetek

Naj bo $\Gamma = (V, E)$ graf. Kvadratni graf Γ^2 grafa Γ je graf z množico vozlišč $V(\Gamma^2) = V$, v katerem sta dve vozlišči sosednji, če je njuna razdalja v grafu Γ največ dve. Kvadratni graf hiperkocke Q_n ima določene zanimive lastnosti. Tako je npr. visoko simetričen in vsepovezan. V tem članku raziskujemo nekatere algebraične lastnosti grafa Q_n^2 . V prvi vrsti pokažemo, da je graf Q_n^2 razdaljno tranzitiven. Dokažemo tudi, da je graf Q_n^2 neprimitiven razdaljno tranzitiven graf natanko takrat, ko je n sodo število. Določimo tudi spekter grafa Q_n^2 . Nazadnje dokažemo: če je $n > 2$ sodo število, potem je Q_n^2 avtomorfen graf, kar pomeni, da je Q_n^2 razdaljno tranzitiven primitiven graf, ki ni ne polni ne povezavni graf.

Ključne besede: Kvadrat grafa, razdaljno tranzitiven graf, hiperkocka, grupa avtomorfizmov, Johnsonov graf, avtomorfni graf.

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