


# Partial-dual Euler-genus distributions for bouquets with small Euler genus\*

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## Abstract

For an arbitrary ribbon graph  $G$ , the partial-dual Euler-genus polynomial of  $G$  is a generating function that enumerates partial duals of  $G$  by Euler genus. When  $G$  is an orientable ribbon graph, the partial-dual orientable genus polynomial of  $G$  is a generating function that enumerates partial duals of  $G$  by orientable genus. Gross, Mansour, and Tucker inaugurated these partial-dual Euler-genus and orientable genus distribution problems in 2020. A bouquet is a one-vertex ribbon graph. Given a ribbon graph  $G$ , its partial-dual Euler-genus polynomial is the same as that of some bouquet; this motivates our focus on bouquets. We obtain the partial-dual Euler-genus polynomials for all the bouquets with Euler genus at most two.

*Keywords:* Ribbon graph, partial dual, Euler-genus polynomial, orientable genus polynomial.

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# Parcialno dualne distribucije Eulerjevega rodu šopkov z majhnim Eulerjevim rodom\*

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## Povzetek

Če je  $G$  poljuben trakovni graf, potem je parcialno dualni polinom Eulerjevega rodu grafa  $G$  rodovna funkcija, ki enumerira parcialne duale grafa  $G$  s pomočjo Eulerjevega roda. V primeru, ko je  $G$  orientabilen trakovni graf, je parcialno dualni polinom orientabilnega rodu grafa  $G$  rodovna funkcija, ki enumerira parcialne duale grafa  $G$  s pomočjo orientabilnega rodu. Gross, Mansour in Tucker so vpeljali distribucijske probleme v zvezi s parcialno dualnim Eulerjevim rodom in orientabilnim rodom leta 2020. Šopek je enotočkovni trakovni graf. Če je dan trakovni graf  $G$ , je njegov polinom parcialno dualnega Eulerjevega rodu enak ustreznemu polinomu nekega šopka; prav to je spodbudilo naše nadaljnje preučevanje šopkov. V članku določimo polinome parcialno dualnega Eulerjevega rodu za vse šopke, katerih Eulerjev rod je največ dva.

*Ključne besede: Trakovni graf, parcialni dual, polinom Eulerjevega rodu, polinom orientabilnega rodu.*

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