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The $L_2(11)$-subalgebra of the Monster algebra

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Abstract

We study a subalgebra V of the Monster algebra, V_{M}, generated by three Majorana axes a_x, a_y and a_z indexed by the $2A$-involutions x, y and z of M, the Monster simple group. We use the notation $V = \ll a_x, a_y, a_z \gg$. We assume that xy is another $2A$-involution and that each of xz, yz and xyz has order 5. Thus a subgroup G of M generated by $\{x, y, z\}$ is a non-trivial quotient of the group $G^{(5,5,5)} = < x, y, z \mid x^2, y^2, (xy)^2, z^2, (xz)^5, (yz)^5, (xyz)^5 >$. It is known that $G^{(5,5,5)}$ is isomorphic to the projective special linear group $L_2(11)$ which is simple, so that G is isomorphic to $L_2(11)$. It was proved by S. Norton that (up to conjugacy) G is the unique $2A$-generated $L_2(11)$-subgroup of M and that $K = C_{\mathrm{M}}(G)$ is isomorphic to the Mathieu group M_{12}. For any pair $\{t, s\}$ of $2A$-involutions, the pair of Majorana axes $\{a_t, a_s\}$ generates the dihedral subalgebra $\ll a_t, a_s \gg$ of V_M, whose structure has been described in . In particular, the subalgebra $\ll a_t, a_s \gg$ contains the Majorana axis a_{tst} by the conjugacy property of dihedral subalgebras. Hence from the structure of its dihedral subalgebras, V coincides with the subalgebra of V_M generated by the set of Majorana axes $\{a_t \mid t \in T\}$, indexed by the 55 elements of the unique conjugacy class T of involutions of $G \cong L_2(11)$. We prove that V is 101-dimensional, linearly spanned by the set $\{ a_t \cdot a_s \mid s, t \in T \}$, and with $C_{V_M}(K) = V \oplus \iota_M$, where ι_M is the identity of V_M. Lastly we present a recent result of Á. Seress proving that V is equal to the algebra of the unique Majorana representation of $L_2(11)$.

Keywords

Monster algebra, Majorana, L2(11)

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The $L_2(11)$-podalgebra algebre Pošasti

Povzetek

V članku študiramo podalgebro V algebre Pošasti, V_M, generirane s tremi Majorana osmi a_x, a_y in a_z indeksirane z $2A$-involucijami x, y in z enostavne grupe M, znane pod imenom Pošast. Uporabljamo zapis $V = \langle\langle a_x, a_y, a_z \rangle\rangle$. Predpostavljamo, da je tudi xy $2A$-involucija in da imajo xz, yz in xyz red 5. Tedaj je podgrupa G grupe M, generirana z $\{x, y, z\}$, netrivialen kvocient grupe $G^{(5,5,5)} = < x, y, z$

$\mid x^2,\ y^2,\ (xy)^2,\ z^2,\ (xz)^5,\ (yz)^5, (xyz)^5 >$. Znano je, da je $G^{(5,5,5)}$ izomorfna projektivni posebni linearni grupi $L_2(11)$, ki je enostavna, tako da je G izomorfna $L_2(11)$. Kot je dokazal S. Norton, je (do konjugiranja natančno) G edina $2A$-generirana $L_2(11)$-podgrupa grupe M in $K = C_{\mathrm{M}}(G)$ je izomorfna Mathieujevi grupi M_{12}. Za poljuben par $\{t,\ s\}$ $2A$-involucij par Majorana osi $\{a_t,\ a_s\}$ generira diedrsko podalgebro $\ll a_t, a_s \gg$ od V_M, katere struktura je bila opisana v . Posebej, podalgebra $\ll a_t, a_s \gg$ vsebuje Majorana osi a_{tst} po konjugacijskih lastnostih diedrskih podalgebr. Torej zaradi strukture njenih diedrskih podalgebr, V sovpada s podalgebro od V_{M}, generirano z množico Majorana osi $\{a_t \mid t \in T\}$, indeksirano s 55 elementi edinega konjugacijskega razreda T involucij od $G \cong L_2(11)$. Dokažemo, da je V 101-dimenzionalna, linearno napeta na množico $\{\ a_t \cdot a_s \mid s,\ t \in T\ \}$, in da je $C_{V\mathrm{M}}(K) = V \oplus \iota_{\mathrm{M}}$, kjer je ι_{M} identiteta v V_{M}. Nazadnje predstavimo nedavni rezultat Á. Seressa, ki dokazuje da je V enaka algebri edine Majorana reprezentacije grupe $L_2(11)$.

Ključne besede

Algebra grupe Pošast, Majorana, L2(11)