ISSN 1855-3966 (printed edn.), ISSN 1855-3974 (electronic edn.)
ARS MATHEMATICA CONTEMPORANEA 22 (2022) #P2.09
https://doi.org/10.26493/1855-3974.2577.25d
(Also available at http://amc-journal.eu)

Linkedness of Cartesian products of complete graphs*

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Received 9 March 2021, accepted 26 August 2021, published online 27 May 2022

Abstract

This paper is concerned with the linkedness of Cartesian products of complete graphs. A graph with at least $2k$ vertices is k-*linked* if, for every set of $2k$ distinct vertices organised in arbitrary k pairs of vertices, there are k vertex-disjoint paths joining the vertices in the pairs.

We show that the Cartesian product $K^{d_1+1} \times K^{d_2+1}$ of complete graphs K^{d_1+1} and K^{d_2+1} is $\lfloor (d_1+d_2)/2 \rfloor$-linked for $d_1, d_2 \geq 2$, and this is best possible.

This result is connected to graphs of simple polytopes. The Cartesian product $K^{d_1+1} \times K^{d_2+1}$ is the graph of the Cartesian product $T(d_1) \times T(d_2)$ of a d_1-dimensional simplex $T(d_1)$ and a d_2-dimensional simplex $T(d_2)$. And the polytope $T(d_1) \times T(d_2)$ is a *simple polytope*, a (d_1+d_2)-dimensional polytope in which every vertex is incident to exactly d_1+d_2 edges.

While not every d-polytope is $\lfloor d/2 \rfloor$-linked, it may be conjectured that every simple d-polytope is. Our result implies the veracity of the revised conjecture for Cartesian products of two simplices.

Keywords: k-linked, cyclic polytope, connectivity, dual polytope, linkedness, Cartesian product.

Math. Subj. Class. (2020): 05C40, 52B05

*The authors want to thank the referee for his/her comments, which have certainly helped to improve the presentation of the paper.

†Corresponding author. Guillermo would like to thank the hospitality of Leif Jørgensen, and the Department of Mathematical Sciences at Aalborg University, where this research started.

‡Julien Ugon's research was supported by the ARC discovery project DP180100602.

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ISSN 1855-3966 (tiskana izd.), ISSN 1855-3974 (elektronska izd.)
ARS MATHEMATICA CONTEMPORANEA 22 (2022) #P2.09
https://doi.org/10.26493/1855-3974.2577.25d
(Dostopno tudi na http://amc-journal.eu)

Veznost kartezičnega produkta polnih grafov*

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Prejeto 9. marca 2021, sprejeto 26. avgusta 2021, objavljeno na spletu 27. maja 2022

Povzetek

Ta članek obravnava veznost kartezičnega produkta polnih grafov. Graf z najmanj $2k$ vozlišči je *k-vezan*, če za vsako množico $2k$ različnih vozlišč, razvrščenih v poljubne k pare, obstaja k vozliščno disjunktnih poti, ki ta vozlišča povezujejo v pare.

Dokažemo, da je kartezični produkt $K^{d_1+1} \times K^{d_2+1}$ polnih grafov K^{d_1+1} in K^{d_2+1} $\lfloor (d_1+d_2)/2 \rfloor$-vezan za $d_1, d_2 \geq 2$, in da je to najboljši možni rezultat.

Ta razultat je povezan z grafi enostavnih politopov. Kartezični produkt $K^{d_1+1} \times K^{d_2+1}$ je graf kartezičnega produkta $T(d_1) \times T(d_2)$ d_1-dimenzionalnega simpleksa $T(d_1)$ ter d_2-dimenzionalnega simpleksa $T(d_2)$. Politop $T(d_1) \times T(d_2)$ pa je *enostaven politop*, (d_1+d_2)-dimenzionalen politop, v katerem je vsaka točka incidentna natančno d_1+d_2 povezavam.

Čeprav ni vsak d-politop $\lfloor d/2 \rfloor$-vezan, pa lahko domnevamo, da vsak enostaven d-politop to je. Naš rezultat implicira resničnost revidirane domneve za kartezične produkte dveh simpleksov.

Ključne besede: k-vezan, ciklični politop, povezljivost, dualni politop, vezanost, kartezični produkt.

Math. Subj. Class. (2020): 05C40, 52B05

*Avtorji se zahvaljujejo recenzentu za njegove/njene pripombe, ki so zagotovo pomagale izboljšati predstavitev članka.

†Kontaktni avtor. Guillermo se zahvaljuje za gostoljubnost Leifu Jørgensenu ter Department of Mathematical Sciences na Aalborg University, kjer se je ta raziskava začela.

‡Julien Ugonova raziskava je bila podprta s strani ARC raziskovalnega projekta DP180100602.

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