

On generalized Minkowski arrangements*

Máté Kadlicskó 

*Department of Geometry, Budapest University of Technology,
Egry József utca 1., Budapest, Hungary, 1111*

Zsolt Lángi [†] 

*MTA-BME Morphodynamics Research Group and Department of Geometry,
Budapest University of Technology,
Egry József utca 1., Budapest, Hungary, 1111*

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Abstract

The concept of a Minkowski arrangement was introduced by Fejes Tóth in 1965 as a family of centrally symmetric convex bodies with the property that no member of the family contains the center of any other member in its interior. This notion was generalized by Fejes Tóth in 1967, who called a family of centrally symmetric convex bodies a generalized Minkowski arrangement of order μ for some $0 < \mu < 1$ if no member K of the family overlaps the homothetic copy of any other member K' with ratio μ and with the same center as K' . In this note we prove a sharp upper bound on the total area of the elements of a generalized Minkowski arrangement of order μ of finitely many circular disks in the Euclidean plane. This result is a common generalization of a similar result of Fejes Tóth for Minkowski arrangements of circular disks, and a result of Böröczky and Szabó about the maximum density of a generalized Minkowski arrangement of circular disks in the plane. In addition, we give a sharp upper bound on the density of a generalized Minkowski arrangement of homothetic copies of a centrally symmetric convex body.

Keywords: Arrangement, Minkowski arrangement, density, homothetic copy.

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[†]Corresponding author.

E-mail addresses: kadlicsko.mate@gmail.com (Máté Kadlicskó), zlangi@math.bme.hu (Zsolt Lángi)

Posplošeni sestavi Minkowskega*

Máté Kadlicskó 

*Department of Geometry, Budapest University of Technology,
Egry József utca 1., Budapest, Hungary, 1111*

Zsolt Lángi † 

*MTA-BME Morphodynamics Research Group and Department of Geometry,
Budapest University of Technology,
Egry József utca 1., Budapest, Hungary, 1111*

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Povzetek

Pojem sestava Minkowskega je vpeljal Fejes Tóth leta 1965 kot družino središčno simetričnih konveksnih teles z lastnostjo, da noben član te družine ne vsebuje središča nobenega drugega člana v svoji notranjosti. Ta pojem je posplošil Fejes Tóth leta 1967, ko je imenoval družino središčno simetričnih konveksnih teles posplošen sestav Minkowskega reda μ za neki $0 < \mu < 1$, če noben član K te družine ne prekriva homotetične kopije nobenega drugega člana K' z razmerjem μ in z istim središčem, kot ga ima K' . V tem članku dokažemo ostro zgornjo mejo za celotno površino elementov posplošenega sestava Minkowskega reda μ , sestavljenega iz končno mnogo krožnih območij v evklidski ravnini. Ta rezultat je posplošitev podobnega rezultata Fejesa Tótha za sestave Minkowskega iz krožnih območij, kot tudi posplošitev rezultata Böröczkyja in Szabá o maksimalni gostoti posplošenega sestava Minkowskega, sestoječega iz krožnih območij v ravnini. Izpeljemo tudi ostro zgornjo mejo za gostoto posplošenega sestava Minkowskega, sestoječega iz homotetičnih kopij središčno simetričnega konveksnega telesa.

Ključne besede: Sestav, sestav Minkowskega, gostota, homotetična kopija.

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† Kontaktni avtor.

E-poštna naslova: kadlicsko.mate@gmail.com (Máté Kadlicskó), zlangi@math.bme.hu (Zsolt Lángi)