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# Complexity of circulant graphs with non-fixed jumps, its arithmetic properties and asymptotics*

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## Abstract

In the present paper, we investigate a family of circulant graphs with non-fixed jumps

$$G_n = C_{\beta n}(s_1, \ldots, s_k, \alpha_1 n, \ldots, \alpha_\ell n),$$
$$1 \le s_1 < \ldots < s_k < [\frac{\beta n}{2}],\ 1 \le \alpha_1 < \ldots < \alpha_\ell \le [\frac{\beta}{2}].$$

Here $n$ is an arbitrary large natural number and integers $s_1, \ldots, s_k, \alpha_1, \ldots, \alpha_\ell, \beta$ are supposed to be fixed.

First, we present an explicit formula for the number of spanning trees in the graph $G_n$. This formula is a product of $\beta s_k - 1$ factors, each given by the $n$-th Chebyshev polynomial of the first kind evaluated at the roots of some prescribed polynomial of degree $s_k$. Next, we provide some arithmetic properties of the complexity function. We show that the number of spanning trees in $G_n$ can be represented in the form $\tau(n) = p\,n\,a(n)^2$, where $a(n)$ is an integer sequence and $p$ is a given natural number depending on parity of $\beta$ and $n$. Finally, we find an asymptotic formula for $\tau(n)$ through the Mahler measure of the Laurent polynomials differing by a constant from $2k - \sum_{i=1}^{k}(z^{s_i} + z^{-s_i})$.

*Keywords: Spanning tree, circulant graph, Laplacian matrix, Chebyshev polynomial, Mahler measure.*

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# Kompleksnost cirkulantov z nefiksiranimi skoki, njene aritmetične lastnosti in asimptotika*

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**Povzetek**

V tem članku preučujemo družino cirkulantov z nefiksiranimi skoki

$$G_n = C_{\beta n}(s_1, \ldots, s_k, \alpha_1 n, \ldots, \alpha_\ell n),$$
$$1 \le s_1 < \ldots < s_k < [\frac{\beta n}{2}], \; 1 \le \alpha_1 < \ldots < \alpha_\ell \le [\frac{\beta}{2}].$$

Tukaj je $n$ poljubno veliko naravno število, cela števila $s_1, \ldots, s_k, \alpha_1, \ldots, \alpha_\ell, \beta$ pa so fiksirana.

Najprej predstavimo eksplicitno formulo za število vpetih dreves v grafu $G_n$. Ta formula je produkt $\beta s_k - 1$ faktorjev, vsak od njih je podan z $n$-tim Čebiševljevim polinomom prve vrste, evaluiranim pri ničlah nekega predpisanega polinoma stopnje $s_k$. Nadalje predstavimo nekaj aritmetičnih lastnosti kompleksnostne funkcije. Dokažemo, da se da število vpetih dreves v grafu $G_n$ zapisati v obliki $\tau(n) = p\, n\, a(n)^2$, kjer je $a(n)$ celoštevilsko zaporedje, $p$ pa dano naravno število, odvisno od sodosti števil $\beta$ in $n$. Poiščemo še asimptotsko formulo za $\tau(n)$ preko Mahlerjeve mere Laurentovih polinomov, ki se od $2k - \sum_{i=1}^{k}(z^{s_i} + z^{-s_i})$ razlikujejo samo za konstanto.

*Ključne besede: Vpeto drevo, cirkulant, Laplaceova matrika, Čebiševljev polinom, Mahlerjeva mera.*

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