

Paired domination stability in graphs

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Abstract

A set S of vertices in a graph G is a paired dominating set if every vertex of G is adjacent to a vertex in S and the subgraph induced by S contains a perfect matching (not necessarily as an induced subgraph). The paired domination number, $\gamma_{\text{pr}}(G)$, of G is the minimum cardinality of a paired dominating set of G . A set of vertices whose removal from G produces a graph without isolated vertices is called a non-isolating set. The minimum cardinality of a non-isolating set of vertices whose removal decreases the paired domination number is the γ_{pr}^- -stability of G , denoted $\text{st}_{\gamma_{\text{pr}}}^-(G)$. The paired domination stability of G is the minimum cardinality of a non-isolating set of vertices in G whose removal changes the paired domination number. We establish properties of paired domination stability in graphs. We prove that if G is a connected graph with $\gamma_{\text{pr}}(G) \geq 4$, then $\text{st}_{\gamma_{\text{pr}}}^-(G) \leq 2\Delta(G)$ where $\Delta(G)$ is the maximum degree in G , and we characterize the infinite family of trees that achieve equality in this upper bound.

Keywords: Paired domination, paired domination stability.

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Stabilnost parne dominacije v grafih

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Povzetek

Množica S vozlišč grafa G je *parna dominacijska množica*, če je vsako vozlišče grafa G sosedno nekemu vozlišču iz množice S in če podgraf, induciran z množico S , vsebuje popolno prirejanje (ne nujno kot induciran podgraf). *Parno dominacijsko število*, $\gamma_{\text{pr}}(G)$, grafa G je minimalna moč parne dominacijske množice grafa G . Množica vozlišč, katerih odstranitev iz G nam da graf brez izoliranih vozlišč, se imenuje *neizolativna množica*. Minimalna moč neizolativne množice vozlišč, katerih odstranitev zmanjša parno dominacijsko število, je γ_{pr}^- -stabilnost grafa G , označena z $\text{st}_{\gamma_{\text{pr}}}^-(G)$. *Stabilnost parne dominacije* grafa G je minimalna moč neizolativne množice vozlišč iz G , katerih odstranitev spremeni parno dominacijsko število. Določimo lastnosti stabilnosti parne dominacije v grafih. Dokažemo: če je G povezan graf in je $\gamma_{\text{pr}}(G) \geq 4$, potem je $\text{st}_{\gamma_{\text{pr}}}^-(G) \leq 2\Delta(G)$, kjer je $\Delta(G)$ maksimalna stopnja vozlišč v grafu G ; karakteriziramo tudi neskončno družino dreves, za katere v tej zgornji meji velja enakost.

Ključne besede: Parna dominacija, stabilnost parne dominacije.

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