

# A parametrisation for symmetric designs admitting a flag-transitive, point-primitive automorphism group with a product action\*

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## Abstract

We study  $(v, k, \lambda)$ -symmetric designs having a flag-transitive, point-primitive automorphism group, with  $v = m^2$  and  $(k, \lambda) = t > 1$ , and prove that if  $D$  is such a design with  $m$  even admitting a flag-transitive, point-primitive automorphism group  $G$ , then either:

- (1)  $D$  is a design with parameters  $\left( (2t + s - 1)^2, \frac{2t^2 - (2-s)t}{s}, \frac{t^2 - t}{s^2} \right)$  with  $s \geq 1$  odd, or
- (2)  $G$  does not have a non-trivial product action.

We observe that the parameters in (1), when  $s = 1$ , correspond to Menon designs.

We also prove that if  $D$  is a  $(v, k, \lambda)$ -symmetric design with a flag-transitive, point-primitive automorphism group of product action type with  $v = m^l$  and  $l \geq 2$  then the complement of  $D$  does not admit a flag-transitive automorphism group.

*Keywords:* Symmetric-designs, flag-transitivity, primitive groups, automorphism groups of designs.

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# Parametrizacija simetričnih dizajnov, ki dopuščajo praporno tranzitivno in točkovno primitivno grupo avtomorfizmov s produktnim delovanjem\*

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## Povzetek

Preučujemo  $(v, k, \lambda)$ -simetrične dizajne, ki imajo praporno tranzitivno in točkovno primitivno grupo avtomorfizmov, kjer je  $v = m^2$  in  $(k, \lambda) = t > 1$ , ter dokažemo, da če je  $D$  dizajn s sodim  $m$ , ki dopušča praporno tranzitivno in točkovno primitivno grupo avtomorfizmov  $G$ , potem velja bodisi:

- (1)  $D$  je dizajn s parametri  $\left( (2t + s - 1)^2, \frac{2t^2 - (2-s)t}{s}, \frac{t^2 - t}{s^2} \right)$ , kjer je  $s \geq 1$  lih, bodisi
- (2)  $G$  nima netrivialnega produktnega delovanja.

Opazimo, da parametri v (1), kadar je  $s = 1$ , ustrezajo Menonovim dizajnom.

Dokažemo tudi, da če je  $D(v, k, \lambda)$ -simetričen dizajn s praporno tranzitivno in točkovno primitivno grupo avtomorfizmov s produktnim delovanjem, kjer je  $v = m^l$  in  $l \geq 2$ , potem komplement dizajna  $D$  ne dopušča praporno tranzitivne grupe avtomorfizmov.

*Ključne besede: Simetrični dizajni, praporna tranzitivnost, primitivne grupe, grupa avtomorfizmov dizajnov.*

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