


Nonlinear maps preserving the elementary symmetric functions

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Abstract

Let \mathcal{M}_n be the algebra of all $n \times n$ complex matrices, and for a natural number $2 \leq k \leq n$ denote by $E_k(x)$ the k th elementary symmetric function on the eigenvalues of $x \in \mathcal{M}_n$. For two maps $\varphi, \psi: \mathcal{M}_n \rightarrow \mathcal{M}_n$, one of them being surjective, we prove that if $E_k(\lambda x + y) = E_k(\lambda \varphi(x) + \psi(y))$ for each $\lambda \in \mathbb{C}$ and $x, y \in \mathcal{M}_n$, then $\varphi = \psi$ on \mathcal{M}_n , the common value being a linear map from \mathcal{M}_n into itself. In particular, for $3 \leq k \leq n$ the general form of φ and ψ can be computed explicitly.

Keywords: Elementary symmetric function, nonlinear, preserver.

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Nelinearne preslikave, ki ohranjajo elementarne simetrične funkcije

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Povzetek

Naj bo \mathcal{M}_n algebra vseh kompleksnih matrik velikosti $n \times n$; za naravno število $2 \leq k \leq n$ označimo z $E_k(x)$ k -to elementarno simetrično funkcijo na lastnih vrednostih matrike $x \in \mathcal{M}_n$. Za dve preslikavi $\varphi, \psi: \mathcal{M}_n \rightarrow \mathcal{M}_n$, pri čemer je ena od njiju surjektivna, dokažemo, da, če je $E_k(\lambda x + y) = E_k(\lambda \varphi(x) + \psi(y))$ za vsak $\lambda \in \mathbb{C}$ in $x, y \in \mathcal{M}_n$, velja $\varphi = \psi$ na \mathcal{M}_n , pri čemer je ta skupna vrednost linearna preslikava množice \mathcal{M}_n vase. Posebej, za $3 \leq k \leq n$, lahko splošno obliko φ in ψ izračunamo eksplicitno.

Ključne besede: Elementarne simetrične funkcije, nelinearen, ohranjevalec.

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