

# Enumerating symmetric peaks in non-decreasing Dyck paths\*

Sergi Elizalde 

*Department of Mathematics, Dartmouth College, Hanover, NH, U.S.A.*

Rigoberto Flórez † 

*Department of Mathematical Sciences, The Citadel, Charleston, SC, U.S.A.*

José Luis Ramírez ‡ 

*Departamento de Matemáticas, Universidad Nacional de Colombia, Bogotá, Colombia*

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## Abstract

Local maxima and minima of a Dyck path are called *peaks* and *valleys*, respectively. A Dyck path is *non-decreasing* if the heights ( $y$ -coordinates) of its valleys increase from left to right. A peak is symmetric if it is surrounded by two valleys (or endpoints of the path) at the same height. In this paper we give multivariate generating functions, recurrence relations, and closed formulas to count the number of symmetric and asymmetric peaks in non-decreasing Dyck paths. Finally, we use Riordan arrays to study weakly symmetric peaks, namely those for which the valley preceding the peak is at least as high as the valley following it.

*Keywords:* Non-decreasing Dyck path, symmetric peak, generating function, Riordan array, Fibonacci number.

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*E-mail addresses:* [sergi.elizalde@dartmouth.edu](mailto:sergi.elizalde@dartmouth.edu) (Sergi Elizalde), [rigo.florez@citadel.edu](mailto:rigo.florez@citadel.edu) (Rigoberto Flórez), [jramirezr@unal.edu.co](mailto:jramirezr@unal.edu.co) (José Luis Ramírez)

# Preštevanje simetričnih vrhov nepadajočih Dyckovih poti\*

Sergi Elizalde 

*Department of Mathematics, Dartmouth College, Hanover, NH, U.S.A.*

Rigoberto Flórez † 

*Department of Mathematical Sciences, The Citadel, Charleston, SC, U.S.A.*

José Luis Ramírez ‡ 

*Departamento de Matemáticas, Universidad Nacional de Colombia, Bogotá, Colombia*

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## Povzetek

Lokalni maksimumi in minimumi Dyckovih poti se imenujejo *vrhovi* in *doline*. Dyck-ova pot je *nepadajoča*, če višine ( $y$ -koordinate) njenih dolin naraščajo od leve proti desni. Vrh je simetričen, če je obdan z dvema dolinama (oz. krajiščema poti) na isti višini. V tem članku predstavimo rodovne funkcije več spremenljivk, rekurzivne relacije in zaprete formule za določanje števila simetričnih in asimetričnih vrhov nepadajočih Dyckovih poti. Uporabimo tudi Riordanove matrike za študij šibko simetričnih vrhov, to je tistih, pri katerih je dolina pred naslednjim vrhom najmanj na taki višini kot dolina, ki temu vrhu sledi.

*Ključne besede:* *Nepadajoča Dyckova pot, simetričen vrh, rodovna funkcija, Riordanova matrika, Fibonaccijevo število.*

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†Kontaktni avtor. Avtor je bil delno podprt s strani Citadel Foundation, Charleston, SC.

‡Avtor je bil delno podprt s strani Universidad Nacional de Colombia, projekt št. 46240.

*E-poštni naslovi:* [sergi.elizalde@dartmouth.edu](mailto:sergi.elizalde@dartmouth.edu) (Sergi Elizalde), [rigo.florez@citadel.edu](mailto:rigo.florez@citadel.edu) (Rigoberto Flórez), [jramirezr@unal.edu.co](mailto:jramirezr@unal.edu.co) (José Luis Ramírez)