

# Modifications of the Floyd-Warshall algorithm with nearly quadratic expected-time\*

Andrej Brodnik <sup>†</sup> 

*University of Primorska, UP FAMNIT, Glagoljaška 8, 6000 Koper, Slovenia*  
*University of Primorska, UP IAM, Muzejski trg 2, 6000 Koper, Slovenia*  
*University of Ljubljana, UL FRI, Večna pot 113, 1000 Ljubljana, Slovenia*

Marko Grgurovič

*University of Primorska, UP FAMNIT, Glagoljaška 8, 6000 Koper, Slovenia*  
*University of Primorska, UP IAM, Muzejski trg 2, 6000 Koper, Slovenia*

Rok Požar <sup>‡</sup> 

*University of Primorska, UP FAMNIT, Glagoljaška 8, 6000 Koper, Slovenia*  
*University of Primorska, UP IAM, Muzejski trg 2, 6000 Koper, Slovenia*  
*IMFM, Jadranska 19, 1000 Ljubljana, Slovenia*

Received 25 October 2020, accepted 10 April 2021, published online 25 November 2021

---

## Abstract

The paper describes two relatively simple modifications of the well-known Floyd-Warshall algorithm for computing all-pairs shortest paths. A fundamental difference of both modifications in comparison to the Floyd-Warshall algorithm is that the relaxation is done in a smart way. We show that the expected-case time complexity of both algorithms is  $O(n^2 \log^2 n)$  for the class of complete directed graphs on  $n$  vertices with arc weights selected independently at random from the uniform distribution on  $[0, 1]$ . Theoretically best known algorithms for this class of graphs are all based on Dijkstra's algorithm and obtain a better expected-case bound. However, by conducting an empirical evaluation we prove that our algorithms are at least competitive in practice with best know algorithms and, moreover, outperform most of them. The reason for the practical efficiency of the presented algorithms is the absence of use of priority queue.

---

\*A preliminary version of this work has been published in Shortest Path Solvers: From Software to Wetware, volume 32 of Emergence, Complexity and Computation (2018). The authors would like to thank the reviewer for excellent comments that substantially improved the quality of the paper.

<sup>†</sup>This work is sponsored in part by the Slovenian Research Agency (research program P2-0359 and research projects J1-2481, J2-2504, and N2-0171).

<sup>‡</sup>Corresponding author. This work is supported in part by the Slovenian Research Agency (research program P1-0285 and research projects N1-0062, J1-9110, J1-9187, J1-1694, N1-0159, J1-2451).

*Keywords: All-pairs shortest paths, probabilistic analysis.*

*Math. Subj. Class. (2020): 05C85, 68W40*

---

# Spremembi Floyd-Warshallovega algoritma s skoraj kvadratičnim pričakovanim časom\*

Andrej Brodnik † 

*University of Primorska, UP FAMNIT, Glagoljaška 8, 6000 Koper, Slovenia*  
*University of Primorska, UP IAM, Muzejski trg 2, 6000 Koper, Slovenia*  
*University of Ljubljana, UL FRI, Večna pot 113, 1000 Ljubljana, Slovenia*

Marko Grgurovič

*University of Primorska, UP FAMNIT, Glagoljaška 8, 6000 Koper, Slovenia*  
*University of Primorska, UP IAM, Muzejski trg 2, 6000 Koper, Slovenia*

Rok Požar ‡ 

*University of Primorska, UP FAMNIT, Glagoljaška 8, 6000 Koper, Slovenia*  
*University of Primorska, UP IAM, Muzejski trg 2, 6000 Koper, Slovenia*  
*IMFM, Jadranska 19, 1000 Ljubljana, Slovenia*

Prejeto 25. oktobra 2020, sprejeto 10. aprila 2021, objavljeno na spletu 25. novembra 2021

## Povzetek

V članku sta opisani dve relativno preprosti spremembi dobro znanega Floyd-Warshallovega algoritma za izračun najkrajših poti med vsemi pari vozlišč. Bistvena razlika obeh sprememb v primerjavi s Floyd-Warshallovim algoritmom je v tem, da se sproščanje izvede na pameten način. Pokažemo, da je pričakovana časovna zahtevnost obeh algoritmov  $O(n^2 \log^2 n)$  za razred polnih usmerjenih grafov na  $n$  vozliščih, pri čemer so uteži na povezavah izbrane neodvisno z vrednostmi enakomerno porazdeljene naključne spremenljivke na intervalu  $[0, 1]$ . Teoretično najboljši algoritmi za ta razred grafov temeljijo na Dijkstrovem algoritmu in dosežejo boljšo mejo v pričakovanem primeru. Kljub temu z empiričnim ovrednotenjem pokažemo, da sta naša algoritma v praksi vsaj konkurenčna z najbolj znanimi algoritmi in poleg tega prekašata večino od njih. Razlog za praktično učinkovitost predstavljenih algoritmov je odsotnost uporabe vrste s prednostjo.

\*Preliminarna različica tega dela je bila objavljena v 'Shortest Path Solvers: From Software to Wetware, volume 32 of Emergence, Complexity and Computation (2018)'. Avtorji bi se radi zahvalili recenzentu za odlične komentarje, ki so izdatno izboljšali kvaliteto tega članka.

†To delo je delno financirano s strani Javne agencije za raziskovalno dejavnost Republike Slovenije (raziskovalni program P2-0359 ter raziskovalni projekti J1-2481, J2-2504 in N2-0171).

‡Kontaktirani avtor. To delo je delno financirano s strani Javne agencije za raziskovalno dejavnost Republike Slovenije (raziskovalni program P1-0285 ter raziskovalni projekti N1-0062, J1-9110, J1-9187, J1-1694, N1-0159 in J1-2451).

*Ključne besede: Najkrajše poti med vsemi pari vozlišč, verjetnostna analiza.*

*Math. Subj. Class. (2020): 05C85, 68W40*

---