

Characterization of a family of rotationally symmetric spherical quadrangulations

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Abstract

A *spherical quadrangulation* is an embedding of a graph G in the sphere in which each facial boundary walk has length four. Vertices that are not of degree four in G are called *curvature vertices*. In this paper we classify all spherical quadrangulations with n -fold rotational symmetry ($n \geq 3$) that have minimum degree 3 and the least possible number of curvature vertices, and describe all such spherical quadrangulations in terms of nets of quadrilaterals. The description reveals that such rotationally symmetric quadrangulations necessarily also have a pole-exchanging symmetry.

Keywords: Quadrangulation, spherical quadrangulation, rotational symmetry.

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Karakterizacija družine rotacijsko simetričnih sferičnih kvadrangulacij

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Povzetek

Sferična kvadrangulacija je vložitev grafa G v sfero, pri kateri ima robni obhod vsakega lica dolžino štiri. Vozlišča grafa G , ki nimajo stopnje štiri, se imenujejo *vozljišča ukrivljenosti*. V tem članku klasificiramo vse sferične kvadrangulacije z n -kratno rotacijsko simetrijo ($n \geq 3$), ki imajo minimalno stopnjo 3 in najmanjše možno število vozlišč ukrivljenosti; opišemo tudi vse takšne sferične kvadrangulacije s četverkotniškimi mrežami. Ta opis razkrije, da imajo vse takšne rotacijsko simetrične kvadrangulacije tudi simetrijo, ki zamenja pole.

Ključne besede: Kvadrangulacija, sferična kvadrangulacija, rotacijska simetrija.

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