

General d -position sets*

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Abstract

The general d -position number $\text{gp}_d(G)$ of a graph G is the cardinality of a largest set S for which no three distinct vertices from S lie on a common geodesic of length at most d . This new graph parameter generalizes the well studied general position number. We first give some results concerning the monotonic behavior of $\text{gp}_d(G)$ with respect to the suitable values of d . We show that the decision problem concerning finding $\text{gp}_d(G)$ is NP-complete for any value of d . The value of $\text{gp}_d(G)$ when G is a path or a cycle is computed and a structural characterization of general d -position sets is shown. Moreover, we present some relationships with other topics including strong resolving graphs and dissociation sets. We finish our exposition by proving that $\text{gp}_d(G)$ is infinite whenever G is an infinite graph and d is a finite integer.

Keywords: General d -position sets, dissociation sets, strong resolving graphs, computational complexity, infinite graphs.

Math. Subj. Class.: 05C12, 05C63, 05C69

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Splošne d -položajne množice*

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Povzetek

Splošno d -položajno število $gp_d(G)$ grafa G definiramo kot kardinalnost največje množice S , za katero velja, da nobene tri njene različne točke ne ležijo na skupni geodetski dolžini največ d . Ta novi grafovski parameter predstavlja posplošitev dobro raziskanega splošnega položajnega števila. Najprej podamo nekaj rezultatov v zvezi z monotonim obnašanjem parametra $gp_d(G)$ glede na ustrezne vrednosti d . Pokažemo, da je odločitveni problem iskanja $gp_d(G)$ NP-poln za vsako vrednost d . Izračunamo vrednost $gp_d(G)$ v primerih, ko je G pot ali cikel, in prikažemo strukturno karakterizacijo splošnih d -položajnih množic. Predstavimo tudi nekatere navezave na druge teme, vključno s močnimi razrešljivimi grafi in razdružitvenimi množicami. Razlago zaključimo z dokazom, da je v primeru, ko je G neskončen graf, d pa končno celo število, vrednost parametra $gp_d(G)$ neskončna.

Ključne besede: Splošne d -položajne množice, razdružitvene množice, močni razrešljivi grafi, računaska zahtevnost, neskončni grafi.

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