


# Coarse distinguishability of graphs with symmetric growth\*

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Received 5 June 2020, accepted 25 March 2021, published online 19 august 2021

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## Abstract

Let  $X$  be a connected, locally finite graph with symmetric growth. We prove that there is a vertex coloring  $\phi: X \rightarrow \{0, 1\}$  and some  $R \in \mathbb{N}$  such that every automorphism  $f$  preserving  $\phi$  is  $R$ -close to the identity map; this can be seen as a coarse geometric version of symmetry breaking. We also prove that the infinite motion conjecture is true for graphs where at least one vertex stabilizer  $S_x$  satisfies the following condition: for every non-identity automorphism  $f \in S_x$ , there is a sequence  $x_n$  such that  $\lim d(x_n, f(x_n)) = \infty$ .

*Keywords:* Graph, coloring, distinguishing, coarse, growth, symmetry.

*Math. Subj. Class.:* 05C15, 51F30

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\*The authors are partially supported by the Program for the Promotion of International Research by Ritsumeikan University and grants FEDER/Ministerio de Ciencia, Innovación y Universidades/AEI/MTM2017-89686-P; and Xunta de Galicia/ED431C 2019/10. We would also like to thank the anonymous referee for a careful reading of the paper.

†Corresponding author. Part of this work was carried out during the tenure of a Canon Foundation in Europe Research Fellowship by B.L.

‡H.N. is partly supported by JSPS KAKENHI Grant Number 17K14195 and 20K03620.

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# Groba ločljivost grafov s simetrično rastjo\*

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Prejeto 5. junija 2020, sprejeto 25. marca 2021, objavljeno na spletu 19. avgusta 2021

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## Povzetek

Naj bo  $X$  povezan, lokalno končen graf s simetrično rastjo. Dokažemo, da obstaja barvanje točk  $\phi: X \rightarrow \{0, 1\}$  in neko število  $R \in \mathbb{N}$ , za katerega velja, da se vsak avtomorfizem  $f$ , ki ohranja  $\phi$ , v vsaki točki razlikuje od identitete največ za  $R$ ; to lahko imamo za grobo geometrijsko različico zloma simetrije. Dokažemo tudi, da je domneva o neskončnem gibanju resnična za grafe, pri katerih najmanj en točkovni stabilizator  $S_x$  zadošča naslednjemu pogoju: za vsak neidentični avtomorfizem  $f \in S_x$  obstaja zaporedje  $x_n$ , za katerega velja  $\lim d(x_n, f(x_n)) = \infty$ .

*Ključne besede: Graf, barvanje, razlikovanje, grob, rast, simetrija.*

*Math. Subj. Class.: 05C15, 51F30*

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\*Avtorji so delno podprti s strani Programa za promocijo mednarodnega raziskovanja Univerze Ritsumeikan in nepovratnih sredstev FEDER/Ministerio de Ciencia, Innovación y Universidades/AEI/MTM2017-89686-P; ter s strani Xunta de Galicia/ED431C 2019/10. Radi bi se zahvalili tudi anonimnemu recenzentu za skrbno branje članka.

<sup>†</sup>Kontaktni avtor. Del te raziskave je B.L. opravil s podporo Canon Foundation in Europe Research Fellowship.

<sup>‡</sup>H.N. je deloma podprt s strani nepovratnih sredstev JSPS KAKENHI št. 17K14195 in 20K03620.

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