

Tight relative t -designs on two shells in hypercubes, and Hahn and Hermite polynomials*

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Abstract

Relative t -designs in the n -dimensional hypercube \mathcal{Q}_n are equivalent to weighted regular t -wise balanced designs, which generalize combinatorial t - (n, k, λ) designs by allowing multiple block sizes as well as weights. Partly motivated by the recent study on tight Euclidean t -designs on two concentric spheres, in this paper we discuss tight relative t -designs in \mathcal{Q}_n supported on two shells. We show under a mild condition that such a relative t -design induces the structure of a coherent configuration with two fibers. Moreover, from this structure we deduce that a polynomial from the family of the Hahn hypergeometric orthogonal polynomials must have only integral simple zeros. The Terwilliger algebra is the main tool to establish these results. By explicitly evaluating the behavior of the zeros of the Hahn polynomials when they degenerate to the Hermite polynomials under an appropriate limit

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process, we prove a theorem which gives a partial evidence that the non-trivial tight relative t -designs in \mathcal{Q}_n supported on two shells are rare for large t .

Keywords: Relative t -design, association scheme, coherent configuration, Terwilliger algebra, Hahn polynomial, Hermite polynomial.

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Tesni relativni t -dizajni na dveh lupinah hiperkock, ter Hahnovi in Hermiteovi polinomi*

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Povzetek

Relativni t -dizajni v n -dimenzionalni hiperkocki Q_n so ekvivalentni uteženim regularnim t -kratnim uravnoveženim dizajnom, ki posplošujejo kombinatorne t - (n, k, λ) dizajne na ta način, da dovoljujejo večkratne velikosti blokov, pa tudi uteži. Motivirani tudi z nedavno študijo o tesnih evklidskih t -dizajnih na dveh koncentričnih sferah, v tem članku obravnavamo tesne relativne t -dizajne v hiperkocki Q_n , podprte na dveh lupinah. Pokažemo, da, pod blagim pogojem, velja, da takšen relativen t -dizajn inducira strukturo koherentne konfiguracije z dvema vlaknoma. Iz te strukture sklepamo tudi, da mora imeti polinom iz družine Hahnovih hipergeometrijskih ortogonalnih polinomov samo celoštevilске enostavne ničle. Glavno orodje pri izpeljavi teh rezultatov je Terwilligerjeva algebra. Z eksplisitno oceno vedenja ničel Hahnovih polinomov, ki pri ustreznem limitnem procesu degenerirajo v Hermiteove polinome, dokažemo izrek, ki deloma dokazuje, da so netrivialni tesni relativni t -dizajni v Q_n , podprti na dveh lupinah, za velike t redki.

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Ključne besede: Relativni t -dizajn, asociativna shema, koherentna konfiguracija, Terwilligerjeva algebra, Hahnov polinom, Hermiteov polinom.

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