

Density results for Graovac-Pisanski's distance number

Lowell Abrams 

*University Writing Program and Department of Mathematics,
The George Washington University, Washington, DC 20052, USA*

Lindsey-Kay Lauderdale *

Department of Mathematics, Towson University, Towson, MD 21252, USA

Received 2 June 2020, accepted 13 May 2021, published online 30 October 2021

Abstract

The sum of distances between every pair of vertices in a graph G is called the *Wiener index* of G . This graph invariant was initially utilized to predict certain physico-chemical properties of organic compounds. However, the Wiener index of G does not account for any of its symmetries, which are also known to effect these physico-chemical properties. Graovac and Pisanski modified the Wiener index of G to measure the average distance each vertex is displaced under the elements of the symmetry group of G ; we call this the *Graovac-Pisanski (GP) distance number* of G . In this article, we prove that the set of all GP distance numbers of graphs with isomorphic symmetry groups is dense in a half-line. Moreover, for each finite group Γ and each rational number q within this half-line, we present a construction for a graph whose GP distance number is q and whose symmetry group is isomorphic to Γ . This construction results in graphs whose vertex orbits are not connected; we also consider an analogous construction which ensures that all vertex orbits are connected.

Keywords: Wiener index, distance number, Graovac-Pisanski index, graph automorphism group, chemical graph theory.

Math. Subj. Class.: 05C12, 05C25, 05C35, 05C92

*Corresponding author. The author is the Jess and Mildred Fisher Endowed Professor of Mathematics in the Fisher College of Science and Mathematics at Towson University and is partially supported by this endowment.

E-mail addresses: labrams@gwu.edu (Lowell Abrams), llauderdale@towson.edu (Lindsey-Kay Lauderdale)

Rezultati o gostoti vrednosti za razdaljno število Graovac-Pisanskega

Lowell Abrams 

*University Writing Program and Department of Mathematics,
The George Washington University, Washington, DC 20052, USA*

Lindsey-Kay Lauderdale *

Department of Mathematics, Towson University, Towson, MD 21252, USA

Prejeto 2. junija 2020, sprejeto 13. maja 2021, objavljeno na spletu 30. oktobra 2021

Povzetek

Vsota razdalj med vsemi pari točk v grafu G se imenuje *Wienerjev indeks* grafa G . Ta grafovski invarianta se je najprej uporabljala za napovedovanje določenih fizikalno-kemijskih lastnosti organskih spojin. Vendar Wienerjev indeks grafa G ne upošteva nobene od njegovih simetrij, za katere pa je prav tako znano, da vplivajo na te fizikalno-kemijske lastnosti. Graovac in Pisanski sta modificirala Wienerjev indeks grafa G tako, da meri povprečno razdaljo, za katero je vsaka točka predstavljena, če nanjo delujejo elementi grupe simetrij grafa G ; tako spremenjenemu indeksu pravimo *razdaljno število Graovac-Pisanskega* za graf G . V tem članku dokažemo, da je množica vseh razdaljnih števil Graovac-Pisanskega za grafe z izomorfnimi simetrijskimi skupami gosta na določenem poltraku. Poleg tega, za vsako končno grupo Γ in vsako racionalno število q s tega poltraka predstavimo konstrukcijo grafa, katerega razdaljno število Graovac-Pisanskega je q in katerega simetrijska grupa je izomorfná Γ . Ta konstrukcija nam da grafe, katerih točkovne orbite niso povezane; obravnavamo pa tudi analogno konstrukcijo, ki zagotavlja, da so vse točkovne orbite povezane.

Ključne besede: Wienerjev indeks, razdaljno število, indeks Graovac-Pisanskega, grupa avtomorfizmov grafa, kemijska teorija grafov.

Math. Subj. Class.: 05C12, 05C25, 05C35, 05C92

*Kontaktni avtor. Avtorica je Jess and Mildred Fisher Endowed Professor of Mathematics na Fisher College of Science and Mathematics pri Towson University in je deloma podprta s tem financiranjem.

E-poštna naslova: labrams@gwu.edu (Lowell Abrams), llauderdale@towson.edu (Lindsey-Kay Lauderdale)