

Bootstrap percolation via automated conjecturing*

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Abstract

Bootstrap percolation is a simple monotone cellular automaton with a long history in physics, computer science, and discrete mathematics. In k -neighbor bootstrap percolation, a collection of vertices are initially infected. Vertices with at least k infected neighbors subsequently become infected; the process continues until no new vertices become infected. In this paper, we hunt for graphs which can become entirely infected from initial sets which are as small as possible. We use automated conjecture-generating software and a large group lab-based model as a fundamental part of our exploration.

Keywords: Bootstrap percolation, automated conjecturing, graph theory, percolation, cellular automata.

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Zagonska perkolacija s pomočjo avtomatiziranega postavljanja domnev*

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Povzetek

Zagonska perkolacija je preprost monoton celični avtomat z dolgo zgodovino v fiziki, računalništvu in diskretni matematiki. Pri k -sosednji zagonski perkolaciji najprej pobarvamo neko množico vozličš. Nato pobarvamo vozlišča, ki imajo vsaj k pobarvanih sosedov; ta proces se nadaljuje, dokler nobeno novo vozlišče ne postane več pobarvano. V tem članku iščemo grafe, ki jih lahko na ta način pobarvamo, izhajajoč iz nekih osnovnih, kolikor mogoče majhnih, pobarvanih množic. Kot temeljno orodje našega raziskovanja uporabljamo avtomatizirano programsko opremo za postavljanje domnev in velik skupinski laboratorijski model.

Ključne besede: Zagonska perkolacija, avtomatizirano postavljanje domnev, teorija grafov, perkolacija, celični avtomati.

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