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The (non-)existence of perfect codes in Lucas cubes

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Abstract

The *Fibonacci cube* of dimension n, denoted as Γ_n, is the subgraph of the n-cube Q_n induced by vertices with no consecutive 1's. Ashrafi and his co-authors proved the non-existence of perfect codes in Γ_n for $n \geq 4$. As an open problem the authors suggest to consider the existence of perfect codes in generalizations of Fibonacci cubes. The most direct generalization is the family $\Gamma_n(1^s)$ of subgraphs induced by strings without 1^s as a substring where $s \geq 2$ is a given integer. In a precedent work we proved the existence of a perfect code in $\Gamma_n(1^s)$ for $n = 2^p - 1$ and $s \geq 3.2^{p-2}$ for any integer $p \geq 2$. The Lucas cube Λ_n is obtained from Γ_n by removing vertices that start and end with 1. Very often the same problems are studied on Fibonacci cubes and Lucas cube. In this note we prove the non-existence of perfect codes in Λ_n for $n \geq 4$ and prove the existence of perfect codes in some generalized Lucas cube $\Lambda_n(1^s)$.

Keywords: Error correcting codes, perfect code, Fibonacci cube.

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O (ne-)obstoju popolnih kod v Lucasovih kockah

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Povzetek

Fibonaccijeva kocka dimenzije n, označena z Γ_n, je podgraf n-kocke Q_n, ki je induciran z vozlišči brez zaporednih 1. Ashrafi in njegovi soavtorji so dokazali neobstoj popolnih kod v Γ_n za $n \geq 4$. Kot odprt problem predlagajo obravnavo obstoja popolnih kod v posplošitvah Fibonaccijevih kock. Najbolj neposredna posplošitev je družina $\Gamma_n(1^s)$ podgrafov, induciranih z nizi, ki so brez 1^s kot podniza, pri čemer je $s \geq 2$ dano celo število. V prejšnjem članku smo dokazali obstoj popolne kode v $\Gamma_n(1^s)$ za $n = 2^p - 1$ in $s \geq 3.2^{p-2}$ za vsako celo število $p \geq 2$. Lucasovo kocko Λ_n dobimo iz Γ_n z odstranitvijo vozlišč, ki se začnejo in končajo z 1. Pogosto so isti problemi preučevani na Fibonaccijevih kockah in na Lucasovi kocki. V tem kratkem članku dokažemo neobstoj popolnih kod v Λ_n za $n \geq 4$ in dokažemo eksistenco popolnih kod v nekaterih posplošenih Lucasovih kockah $\Lambda_n(1^s)$.

Ključne besede: Kode, ki popravljajo napake, popolna koda, Fibonaccijeva kocka.

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