

# Oriented area as a Morse function on polygon spaces\*

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## Abstract

We study polygon spaces arising from planar configurations of necklaces with some of the beads fixed and some of the beads sliding freely. These spaces include configuration spaces of flexible polygons and some other natural polygon spaces. We characterise critical points of the oriented area function in geometric terms and give a formula for the Morse indices. Thus we obtain a generalisation of isoperimetric theorems for polygons in the plane.

*Keywords:* Flexible polygons, configuration spaces, Morse index, critical points.

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# Orientirana ploščina kot Morseova funkcija na poligonskih prostorih\*

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## Povzetek

Preučujemo poligonske prostore, ki jih dobimo iz ravninskih konfiguracij ogrlic, ki imajo nekaj kroglic pričvrščenih, nekaj pa prosto drsečih. Ti prostori vsebujejo konfiguracyjske prostore gibljivih poligonov, pa tudi nekaj drugih naravnih poligonskih prostorov. Opredelimo kritične točke orientirane ploščinske funkcije z geometrijskimi pojmi in izpeljemo formulo za Morseove indekse. Tako dobimo posplošitev izoperimetričnih izrekov za poligone v ravnini.

*Ključne besede: Gibljivi poligoni, konfiguracyjski prostori, Morseov indeks, kritične točke.*

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