

A generalization of balanced tableaux and marriage problems with unique solutions

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Abstract

We consider families of finite sets that we call flagged and that have been characterized by Chang as being the families of sets that admit unique solutions to Hall's marriage problem and we consider generalizations of Edelman and Greene's balanced tableaux previously investigated by Viard. In this paper, we introduce a natural generalization of Edelman and Greene's balanced tableaux that involves families of sets that satisfy Hall's marriage condition and certain words in $[m]^n$, then prove that flagged families can be characterized by a strong existence condition relating to this generalization. As a consequence of this characterization, we show that the arithmetic mean of the sizes of subclasses of such generalized tableaux is given by a generalization of the hook-length formula.

Keywords: Balanced tableaux, Hall's marriage condition, shelling.

Math. Subj. Class.: 05A20, 05C70, 05E45

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Posplošitev uravnoveženih tablic in problemov porok z enoličnimi rešitvami

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Povzetek

Obravnavamo družine končnih množic, ki jih imenujemo *označene z zastavico*; Chang jih je karakteriziral kot družine množic, ki imajo enolične rešitve Hallovega problema porok. Obravnavamo tudi posplošitve Edelmanovih in Greeneovih tablic, ki jih je obravnaval že Viard. V tem članku vpeljemo naravno posplošitev Edelmanovih in Greeneovih uravnoveženih tablic, ki vključuje družine množic, ki zadoščajo Hallovemu pogoju porok in določene besede v $[m]^n$, nato pa pokažemo, da se da z zastavicami označene družine karakterizirati z močnim eksistenčnim pogojem, ki se nanaša na to posplošitev. Na osnovi te karakterizacije pokažemo, da se da aritmetično sredino velikosti podrazredov teh posplošenih tablic izraziti s posplošitvijo formule za dolžino kljuk.

Ključne besede: Uravnovežene tablice, Hallov pogoj v zvezi s problemom porok, granatiranje.

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