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Maximal order group actions on Riemann surfaces

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Abstract

A natural problem is to determine, for each value of the integer $g \geq 2$, the largest order of a group that acts on a Riemann surface of genus g. Let $N(g)$ (respectively $M(g)$) be the largest order of a group of automorphisms of a Riemann surface of genus $g \geq 2$ preserving the orientation (respectively possibly reversing the orientation) of the surface.

The basic inequalities comparing $N(g)$ and $M(g)$ are $N(g) \leq M(g) \leq 2N(g)$. There are well-known families of extended Hurwitz groups that provide an infinite number of integers g satisfying $M(g) = 2N(g)$. It is also easy to see that there are solvable groups which provide an infinite number of such examples.

We prove that, perhaps surprisingly, there are an infinite number of integers g such that $N(g) = M(g)$. Specifically, if p is a prime satisfying $p \equiv 1 \pmod 6$ and $g = 3p + 1$ or $g = 2p + 1$, there is a group of order $24(g-1)$ that acts on a surface of genus g preserving the orientation of the surface. For all such values of g larger than a fixed constant, there are no groups with order larger than $24(g-1)$ that act on a surface of genus g.

Keywords: Riemann surface, genus, group action, NEC group, strong symmetric genus.

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Delovanja grup maksimalnega reda na Riemannovih ploskvah

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Povzetek

Kako določiti, za vsako vrednost celega števila $g \geq 2$, največji red grupe, ki deluje na Riemannovi ploskvi rodu g, je zelo naraven problem. Naj bo $N(g)$ največji red grupe avtomorfizmov Riemannove ploskve rodu $g \geq 2$, ki ohranjajo orientacijo ploskve, $M(g)$ pa tistih, ki orientacijo morda obrnejo.

Osnovne neenakosti, ki primerjajo $N(g)$ in $M(g)$, so: $N(g) \leq M(g) \leq 2N(g)$. Dobro znane so družine razširjenih Hurwitzevih grup, iz katerih dobimo neskončno mnogo celih števil g, ki zadoščajo enakosti $M(g) = 2N(g)$. Lahko je tudi videti, da obstajajo rešljive grupe, iz katerih dobimo neskončno mnogo takih primerov.

Dokažemo, kar morda preseneča, da obstaja neskončno mnogo celih števil g, za katera je $N(g) = M(g)$. V primeru, da je p praštevilo, ki zadošča $p \equiv 1 \pmod 6$ in $g = 3p + 1$ ali $g = 2p + 1$, obstaja grupa reda $24(g - 1)$, ki deluje na neki ploskvi rodu g, pri čemer ohranja njeno orientacijo. Za vse vrednosti g, ki so večje od neke fiksne konstante, ne obstajajo grupe z redom večjim od $24(g - 1)$, ki bi delovale na ploskvi rodu g.

Ključne besede: Riemannova ploskev, rod, delovanje grupe, NEC grupa, močni simetrični rod.

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