


A double Sylvester determinant*

Darij Grinberg [†] 

Drexel University, Korman Center, 15 S 33rd Street, Philadelphia PA, 19104, USA

Received 7 February 2020, accepted 22 August 2020, published online 18 November 2021

Abstract

Given two $(n + 1) \times (n + 1)$ -matrices A and B over a commutative ring, and some $k \in \{0, 1, \dots, n\}$, we consider the $\binom{n}{k} \times \binom{n}{k}$ -matrix W whose entries are $(k + 1) \times (k + 1)$ -minors of A multiplied by corresponding $(k + 1) \times (k + 1)$ -minors of B . Here we require the minors to use the last row and the last column (which is why we obtain an $\binom{n}{k} \times \binom{n}{k}$ -matrix, not a $\binom{n+1}{k+1} \times \binom{n+1}{k+1}$ -matrix). We prove that the determinant $\det W$ is a multiple of $\det A$ if the $(n + 1, n + 1)$ -th entry of B is 0. Furthermore, if the $(n + 1, n + 1)$ -th entries of both A and B are 0, then $\det W$ is a multiple of $(\det A)(\det B)$. This extends a previous result of Olver and the author.

Keywords: Determinant, compound matrix, Sylvester's determinant, polynomials.


Math. Subj. Class. (2020): 15A15, 11C20

*The author would like to thank Christian Krattenthaler, Peter Olver and Victor Reiner for enlightening discussions, and Peter Olver for the joint work that led to this paper. The SageMath computer algebra system has been used for experimentation leading up to some of the results below.

[†]Homepage: <http://www.cip.ifi.lmu.de/~grinberg/>

E-mail address: darijgrinberg@gmail.com (Darij Grinberg)

Dvojna Sylvestrova determinanta*

Darij Grinberg † *Drexel University, Korman Center, 15 S 33rd Street, Philadelphia PA, 19104, USA*

Prejeto 7. februarja 2020, sprejeto 22. avgusta 2020, objavljeno na spletu 18. novembra 2021

Povzetek

Če sta dani dve $(n + 1) \times (n + 1)$ -matriki A in B nad komutativnim kolobarjem in neko število $k \in \{0, 1, \dots, n\}$, obravnavamo $\binom{n}{k} \times \binom{n}{k}$ -matriko W , katere elementi so $(k + 1) \times (k + 1)$ -minorji matrike A , pomnoženi z ustreznimi $(k + 1) \times (k + 1)$ -minorji matrike B . Pri tem zahtevamo, da minorji vsebujejo zadnjo vrstico in zadnji stolpec (zato dobimo $\binom{n}{k} \times \binom{n}{k}$ -matriko, ne pa $\binom{n+1}{k+1} \times \binom{n+1}{k+1}$ -matrike). Dokažemo, da je determinanta $\det W$ večkratnik determinante $\det A$, če je $(n + 1, n + 1)$ -ti element matrike B enak 0. Nadalje dokažemo, da če sta $(n + 1, n + 1)$ -ta elementa tako matrike A kot matrike B enaka 0, potem je $\det W$ večkratnik od $(\det A)(\det B)$. To predstavlja razširitev prejšnjega rezultata Olverja in avtorja tega članka.

Ključne besede: Determinanta, sestavljena matrika, Sylvestrova determinanta, polinomi.

Math. Subj. Class. (2020): 15A15, 11C20

*Avtor bi se rad zahvalil Christianu Krattenthalerju, Petru Olverju in Victorju Reinerju za navdihujoče razprave, ter Petru Olverju za skupno delo, ki je vodilo k temu članku. Računalniški algebrski sistem SageMath je bil uporabljen za eksperimente, ki so vodili k nekaterim od predstavljenih rezultatov.

†Domača stran: <http://www.cip.ifi.lmu.de/~grinberg/>

E-poštni naslov: darijgrinberg@gmail.com (Darij Grinberg)