

Z-oriented triangulations of surfaces

Adam Tyc * 

*Institute of Mathematics, Polish Academy of Sciences,
Śniadeckich 8, 00-656 Warszawa, Poland*

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Abstract

The main objects of the paper are z -oriented triangulations of connected closed 2-dimensional surfaces. A z -orientation of a map is a minimal collection of zigzags which double covers the set of edges. We have two possibilities for an edge – zigzags from the z -orientation pass through this edge in different directions (type I) or in the same direction (type II). Then there are two types of faces in a triangulation: the first type is when two edges of the face are of type I and one edge is of type II and the second type is when all edges of the face are of type II. We investigate z -oriented triangulations with all faces of the first type (in the general case, any z -oriented triangulation can be shredded to a z -oriented triangulation of such type). A zigzag is homogeneous if it contains precisely two edges of type I after any edge of type II. We give a topological characterization of the homogeneity of zigzags; in particular, we describe a one-to-one correspondence between z -oriented triangulations with homogeneous zigzags and closed 2-cell embeddings of directed Eulerian graphs in surfaces. At the end, we give an application to one type of the z -monodromy.

Keywords: Directed Eulerian embedding, triangulation of a surface, zigzag, z -monodromy, z -orientation.

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E-mail address: atyc@impan.pl (Adam Tyc)

Z-orientirane triangulacije ploskev

Adam Tyc * 

*Institute of Mathematics, Polish Academy of Sciences,
Śniadeckich 8, 00-656 Warszawa, Poland*

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Povzetek

Glavni predmet članka so z -orientirane triangulacije povezanih 2-dimenzionalnih ploskev. z -orientacija zemljevida je minimalna kolekcija cikcak črt, ki dvojno pokriva množico povezav. Imamo dva tipa povezav – cikcak črte iz z -orientacije potekajo skozi dano povežavo v različnih smereh (tip I) ali pa v isti smeri (tip II). V skladu s tem obstajata dva tipa lic v triangulaciji: pri prvem tipu sta dve povezavi lica tipa I, ena povezava pa tipa II, pri drugem tipu pa so vse povezave lica tipa II. Preučujemo z -orientirane triangulacije z vsemi lici prvega tipa (v splošnem primeru velja, da se da poljubna z -orientirana triangulacija razrezati na z -orientirano triangulacijo tega tipa). Cikcak črta je homogena, če vsaki povezavi tipa II sledita natanko dve povezavi tipa I. Podamo topološko karakterizacijo homogenosti cikcak črt; posebej, opišemo bijektivno korespondenco med z -orientiranimi triangulacijami s homogenimi cikcak črtami in zaprtimi 2-celičnimi vložitvami usmerjenih Eulerjevih grafov v ploskve. Na koncu predstavimo aplikacijo na en tip z -monodromije.

Ključne besede: Usmerjena evklidska vložitev, triangulacija ploskve, cikcak črta, z -monodromija, z -orientacija.

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E-poštni naslov: atyc@impan.pl (Adam Tyc)