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The expansion of a chord diagram and the Genocchi numbers

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Abstract

A chord diagram E is a set of chords of a circle such that no pair of chords has a common endvertex. Let $v_1, v_2, \dots, v_{2n}$ be a sequence of vertices arranged in clockwise order along a circumference. A chord diagram $\{v_1v_{n+1}, v_2v_{n+2}, \dots, v_nv_{2n}\}$ is called an n-crossing and a chord diagram $\{v_1v_2, v_3v_4, \dots, v_{2n-1}v_{2n}\}$ is called an n-necklace. For a chord diagram E having a 2-crossing $S = \{x_1x_3, x_2x_4\}$, the expansion of E with respect to S is to replace E with $E_1 = (E \setminus S) \cup \{x_2x_3, x_4x_1\}$ or $E_2 = (E \setminus S) \cup \{x_1x_2, x_3x_4\}$. Beginning from a given chord diagram E as the root, by iterating chord expansions in both ways, we have a binary tree whose all leaves are nonintersecting chord diagrams. Let $\mathcal{NCD}(E)$ be the multiset of the leaves. In this paper, the multiplicity of an n-necklace in $\mathcal{NCD}(E)$ is studied. Among other results, it is shown that the multiplicity of an n-necklace generated from an n-crossing equals the Genocchi number when n is odd and the median Genocchi number when n is even.

Keywords: Chord diagram, chord expansion, Genocchi number, Seidel triangle.

Math. Subj. Class.: 05A15, 05A10

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Razplet tetivnega diagrama in Genocchijeva števila

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Povzetek

Tetivni diagram E je množica tetiv kroga, v kateri noben par tetiv nima skupnega krajišča. Naj bo $v_1, v_2, \ldots, v_{2n}$ zaporedje točk, urejenih v smeri urinega kazalca vzdolž oboda kroga. Tetivni diagram $\{v_1v_{n+1}, v_2v_{n+2}, \ldots, v_nv_{2n}\}$ se imenuje n-*križišče*, tetivni diagram $\{v_1v_2, v_3v_4, \ldots, v_{2n-1}v_{2n}\}$ pa je n-*ogrlica*. Naj bo E tetivni diagram, ki ima 2-križišče $S = \{x_1x_3, x_2x_4\}$; potem se zamenjava E z $E_1 = (E \setminus S) \cup \{x_2x_3, x_4x_1\}$ ali z $E_2 = (E \setminus S) \cup \{x_1x_2, x_3x_4\}$ imenuje *razplet* E glede na S. Če začnemo z danim tetivnim diagramom E kot korenom, potem pa delamo tetivne razplete na oba načina, dobimo dvojiško drevo, katerega listi so izključno tetivni diagrami brez križišč. Naj bo $\mathcal{NCD}(E)$ mnogotera množica listov tega drevesa. V tem članku preučujemo večkratnost n-ogrlice v mnogoteri množici $\mathcal{NCD}(E)$. Poleg drugih rezultatov, ki jih dobimo, pokažemo tudi, da je večkratnost n-ogrlice, dobljene iz n-križišča, enaka Genocchijevemu številu, če je n liho število, in sredinskemu Genocchijevemu številu, če je n sodo število.

Ključne besede: Tetivni diagram, tetivni razplet, Genocchijevo število, Seidelov trikotnik.

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