

# Hamilton cycles in primitive vertex-transitive graphs of order a product of two primes – the case $\mathrm{PSL}(2, q^2)$ acting on cosets of $\mathrm{PGL}(2, q)^*$

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## Abstract

A step forward is made in a long standing Lovász problem regarding hamiltonicity of vertex-transitive graphs by showing that every connected vertex-transitive graph of order a product of two primes arising from the group action of the projective special linear group  $\mathrm{PSL}(2, q^2)$  on cosets of its subgroup isomorphic to the projective general linear group  $\mathrm{PGL}(2, q)$  contains a Hamilton cycle.

*Keywords:* Vertex-transitive graph, Hamilton cycle, automorphism group, orbital graph.

*Math. Subj. Class.:* 05C25, 05C45

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# Hamiltonski cikli v primitivnih vozliščno tranzitivnih grafih, katerih red je produkt dveh praštevil – primer, ko $PSL(2, q^2)$ deluje na odsekih grupe $PGL(2, q)^*$

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## Povzetek

Članek prinaša napredek pri dolgo odprti Lovászovi domnevi o hamiltonskosti vozliščno tranzitivnih grafov. V članku je dokazano, da vsak povezan vozliščno tranzitivni graf, katerega red je produkt dveh praštevil, ki izhaja iz grupnega delovanja specialne projektivne linearne grupe  $PSL(2, q^2)$  na odsekih po njeni podgrupi izomorfni splošni projektivni linearni grupi  $PGL(2, q)$ , premore hamiltonski cikel.

*Ključne besede: Vozliščno tranzitivni graf, hamiltonski cikel, avtomorfizem grupe, orbitalni graf.*

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