

# Graphical Frobenius representations of non-abelian groups\*

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Received 14 October 2019, accepted 19 October 2020, published online 18 August 2021

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## Abstract

A group  $G$  has a Frobenius graphical representation (GFR) if there is a simple graph  $\Gamma$  whose full automorphism group is isomorphic to  $G$  acting on the vertices as a Frobenius group. In particular, any group  $G$  with a GFR is a Frobenius group and  $\Gamma$  is a Cayley graph. By very recent results of Spiga, there exists a function  $f$  such that if  $G$  is a finite Frobenius group with complement  $H$  and  $|G| > f(|H|)$  then  $G$  admits a GFR. This paper provides an infinite family of graphs that admit GFRs despite not meeting Spiga's bound. In our construction, the group  $G$  is the Higman group  $A(f, q_0)$  for an infinite sequence of  $f$  and  $q_0$ , having a nonabelian kernel and a complement of odd order.

*Keywords:* Cayley graph, Frobenius group, Suzuki 2-group, Frobenius graphical representation.

*Math. Subj. Class. (2020):* 20B25, 05C25

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\*Support provided by NKFIH-OTKA Grants 114614, 115288 and 119687.

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# Grafovske Frobeniusove predstavitve nekomutativnih grup\*

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Prejeto 14. oktobra 2019, sprejeto 19. oktobra 2020, objavljeno na spletu 18. avgusta 2021

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## Povzetek

Grupa  $G$  ima Frobeniusovo grafovsko predstavitve, če obstaja enostaven graf  $\Gamma$ , katerega polna grupa avtomorfizmov je izomorfna grupi  $G$ , ki deluje na vozliščih kot Frobeniusova grupa. Vsaka grupa  $G$ , ki ima takšno predstavitve, je Frobeniusova grupa in  $\Gamma$  je Cayleyjev graf. Po nedavnih rezultatih Spige, obstaja funkcija  $f$ , za katero velja, da če je  $G$  končna Frobeniusova grupa s komplementom  $H$  in je  $|G| > f(|H|)$ , potem  $G$  dopušča Frobeniusovo grafovsko predstavitve. V tem članku predstavimo neskončno družino grafov, ki dopuščajo Frobeniusovo grafovsko predstavitve, čeprav ne ustrezajo Spigovi meji. V naši konstrukciji je grupa  $G$  Higmanova grupa  $A(f, q_0)$  za neskončno zaporedje funkcij  $f$  in  $q_0$ , ki ima nekomutativno jedro ter komplement lihega reda.

*Ključne besede: Cayleyjev graf, Frobeniusova grupa, Suzukijeva 2-grupa, Frobeniusova grafovska predstavitve.*

*Math. Subj. Class. (2020): 20B25, 05C25*

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\*Podpora s strani NKFIH-OTKA dotacij 114614, 115288 in 119687.

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