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Balancing polyhedra*

Gábor Domokos

MTA-BME Morphodynamics Research Group and Department of Mechanics, Materials and Structures, Budapest University of Technology, Műegyetem rakpart 1-3., Budapest, Hungary, 1111

Flórián Kovács

MTA-BME Morphodynamics Research Group and Department of Structural Mechanics, Budapest University of Technology, Műegyetem rakpart 1-3., Budapest, Hungary, 1111

Zsolt Lángi †

MTA-BME Morphodynamics Research Group and Department of Geometry, Budapest University of Technology, Egry József utca 1., Budapest, Hungary, 1111

Krisztina Regős , Péter T. Varga

MTA-BME Morphodynamics Research Group, Műegyetem rakpart 1-3., Budapest, Hungary, 1111

Dedicated to the memory of John Horton Conway.

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Abstract

We define the mechanical complexity $C(P)$ of a 3-dimensional convex polyhedron P, interpreted as a homogeneous solid, as the difference between the total number of its faces, edges and vertices and of its static equilibria; and the mechanical complexity $C(S,U)$ of primary equilibrium classes $(S,U)^E$ with S stable and U unstable equilibria as the infimum of the mechanical complexity of all polyhedra in that class. We prove that the mechanical complexity of a class $(S,U)^E$ with $S,U > 1$ is the minimum of $2(f + v -$

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†Corresponding author. The author has been supported by the Bolyai Fellowship of the Hungarian Academy of Sciences and partially supported by the UNKP-20-5 New National Excellence Program of the Ministry of Innovation and Technology.

$S - U$) over all polyhedral pairs (f, v), where a pair of integers is called a polyhedral pair if there is a convex polyhedron with f faces and v vertices. In particular, we prove that the mechanical complexity of a class $(S, U)^E$ is zero if and only if there exists a convex polyhedron with S faces and U vertices. We also give asymptotically sharp bounds for the mechanical complexity of the monostatic classes $(1, U)^E$ and $(S, 1)^E$, and offer a complexity-dependent prize for the complexity of the Gömböc-class $(1, 1)^E$.

Keywords: Polyhedron, static equilibrium, monostatic polyhedron, f-vector.

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E-mail addresses: domokos@iit.bme.hu (Gábor Domokos), kovacs.florian@epito.bme.hu (Flórián Kovács), zlangi@math.bme.hu (Zsolt Lángi), regoskriszti@gmail.com (Krisztina Regős), petercobbler@gmail.com (Péter T. Varga)

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Uravnoteženi poliedri*

Gábor Domokos
MTA-BME Morphodynamics Research Group and Department of Mechanics, Materials and Structures, Budapest University of Technology, Műegyetem rakpart 1-3., Budapest, Hungary, 1111

Flórián Kovács
MTA-BME Morphodynamics Research Group and Department of Structural Mechanics, Budapest University of Technology, Műegyetem rakpart 1-3., Budapest, Hungary, 1111

Zsolt Lángi †
MTA-BME Morphodynamics Research Group and Department of Geometry, Budapest University of Technology, Egry József utca 1., Budapest, Hungary, 1111

Krisztina Regős , Péter T. Varga
MTA-BME Morphodynamics Research Group, Műegyetem rakpart 1-3., Budapest, Hungary, 1111

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Povzetek

Definiramo mehansko kompleksnost $C(P)$ 3-dimenzionalnega konveksnega poliedra P, interpretiranega kot homogeno telo, kot razliko med skupnim številom njegovih lic, povezav in točk ter številom njegovih statičnih ravnovesij; definiramo tudi mehansko kompleksnost $C(S, U)$ primarnih ravnovesnostnih razredov $(S, U)^E$ s S stabilnimi in U nestabilnimi ravnovesji kot natančno spodnjo mejo mehanske kompleksnosti vseh poliedrov v tem razredu. Dokažemo, da je mehanska kompleksnost razreda $(S, U)^E$ pri pogoju

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†Kontaktni avtor. Avtor je bil podprt s strani Bolyai Fellowship of the Hungarian Academy of Sciences in delno podprt s strani UNKP-20-5 New National Excellence Program of the Ministry of Innovation and Technology.

$S, U > 1$ minimum izraza $2(f + v - S - U)$ po vseh poliedrskih parih (f, v), pri čemer se par celih števil imenuje poliedrski par, če obstaja konveksen polieder z f lici in v točkami. Posebej, dokažemo, da je mehanska kompleksnost razreda $(S, U)^E$ enaka nič natanko tedaj, ko obstaja konveksen polieder s S lici in U točkami. Predstavimo tudi asimptotsko ostre meje za mehansko kompleksnost monostatičnih razredov $(1, U)^E$ in $(S, 1)^E$, ter ponudimo od kompleksnosti odvisno nagrado za določitev kompleksnosti Gömböcovega razreda $(1, 1)^E$.

Ključne besede: Polieder, statično ravnovesje, monostatični polieder, f-vektor.

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E-poštni naslovi: domokos@iit.bme.hu (Gábor Domokos), kovacs.florian@epito.bme.hu (Flórián Kovács), zlangi@math.bme.hu (Zsolt Lángi), regoskriszti@gmail.com (Krisztina Regős), petercobbler@gmail.com (Péter T. Varga)