

On the divisibility of binomial coefficients

Sílvia Casacuberta * 

Harvard University, 1 Oxford Street, Cambridge, MA, USA

Received 30 August 2019, accepted 16 August 2020, published online 18 November 2020

Abstract

Shareshian and Woodroffe asked if for every positive integer n there exist primes p and q such that, for all integers k with $1 \leq k \leq n - 1$, the binomial coefficient $\binom{n}{k}$ is divisible by at least one of p or q . We give conditions under which a number n has this property and discuss a variant of this problem involving more than two primes. We prove that every positive integer n has infinitely many multiples with this property.

Keywords: Binomial coefficients, divisibility, primorials.

Math. Subj. Class.: 11B65, 05A10

*I am indebted to my mentor Oscar Mickelin for his guidance throughout this research and to Prof. Russ Woodroffe for correspondence and kind suggestions. This work was carried out during the Research Science Program at MIT in the summer of 2017 and was supported by the Center for Excellence in Education, the MIT Mathematics Department, and the Youth and Science Program of Fundació Catalunya La Pedrera (Barcelona).

E-mail address: scasacubertapuig@college.harvard.edu (Sílvia Casacuberta)

O deljivosti biomskih koeficientov

Sílvia Casacuberta * 

Harvard University, 1 Oxford Street, Cambridge, MA, USA

Prejeto 30. avgusta 2019, sprejeto 16. avgusta 2020, objavljeno na spletu 18. novembra 2020

Povzetek

Shreshian in Woodrooffe sta vprašala, če za poljubno pozitivno celo število n obstajata taki praštevili p in q , da je za vsa cela števila k , za katera velja $1 \leq k \leq n - 1$, binomski koeficient $\binom{n}{k}$ deljiv vsaj z enim od p ali q . V prispevku podamo pogoje, ki jih mora zadoščati n s to lastnostjo, in obravnavamo različice tega problema za več kot dve praštevili. Med drugim dokažemo, da ima vsako naravno število n neskončno mnogo večkratnikov s to lastnostjo.

Ključne besede: Binomski koeficienti, deljivost, primoriele.

Math. Subj. Class.: 11B65, 05A10

*Zahvaljujem se svojemu mentorju Oscarju Mickelinu za njegovo vodenje pri tej raziskavi in prof. Russu Woodrooffu za dopisovanje in prijazne predloge. To delo je bilo izvedeno med raziskovalnim znanstvenim programom na MIT poleti leta 2017, podprli pa so ga "Center for Excellence in Education", "MIT Mathematics Department" ter "Youth and Science Program of Fundació Catalunya La Pedrera (Barcelona)".

E-poštni naslov: scasacubertapuig@college.harvard.edu (Sílvia Casacuberta)