

# Sum-list-colouring of $\theta$ -hypergraphs

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Received 12 August 2019, accepted 10 May 2021, published online 21 February 2022

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## Abstract

Given a hypergraph  $\mathcal{H}$  and a function  $f: V(\mathcal{H}) \rightarrow \mathbb{N}$ , we say that  $\mathcal{H}$  is  $f$ -choosable if there is a proper vertex coloring  $\phi$  of  $\mathcal{H}$  such that  $\phi(v) \in L(v)$  for all  $v \in V(\mathcal{H})$ , where  $L: V(\mathcal{H}) \rightarrow 2^{\mathbb{N}}$  is any assignment of  $f(v)$  colors to a vertex  $v$ . The sum choice number  $\chi_{sc}(\mathcal{H})$  of  $\mathcal{H}$  is defined to be the minimum of  $\sum_{v \in V(\mathcal{H})} f(v)$  over all functions  $f$  such that  $\mathcal{H}$  is  $f$ -choosable. A trivial upper bound on  $\chi_{sc}(\mathcal{H})$  is  $|V(\mathcal{H})| + |\mathcal{E}(\mathcal{H})|$ . The class  $\Gamma_{sc}$  of hypergraphs that achieve this bound is induced hereditary. We analyze some properties of hypergraphs in  $\Gamma_{sc}$  as well as properties of hypergraphs in the class of forbidden hypergraphs for  $\Gamma_{sc}$ . We characterize all  $\theta$ -hypergraphs in  $\Gamma_{sc}$ , which leads to the characterization of all  $\theta$ -hypergraphs that are forbidden for  $\Gamma_{sc}$ .

*Keywords:* Hypergraphs, sum-list-colouring,  $\theta$ -hypergraphs.

*Math. Subj. Class. (2020):* 05C15, 05C65

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## Vsotno seznamsko barvanje $\theta$ -hipergrafov

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Prejeto 12. avgusta 2019, sprejeto 10. maja 2021, objavljeno na spletu 21. februarja 2022

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### Povzetek

Če sta dana hipergraf  $\mathcal{H}$  in funkcija  $f: V(\mathcal{H}) \rightarrow \mathbb{N}$ , pravimo, da je  $\mathcal{H}$   $f$ -izbiren, če obstaja pravilno vozliščno barvanje  $\phi$ , pri katerem je  $\phi(v) \in L(v)$  za vsa vozlišča  $v \in V(\mathcal{H})$ , kjer je  $L: V(\mathcal{H}) \rightarrow 2^{\mathbb{N}}$  poljubna dodelitev  $f(v)$  barv vozlišča  $v$ . Vsotno izbirni indeks  $\chi_{sb}(\mathcal{H})$  hipergrafa  $\mathcal{H}$  je definiran kot minimum vsote  $\sum_{v \in V(\mathcal{H})} f(v)$  po vseh funkcijah  $f$ , za katere je  $\mathcal{H}$   $f$ -izbiren. Trivialna zgornja meja za  $\chi_{sb}(\mathcal{H})$  je  $|V(\mathcal{H})| + |\mathcal{E}(\mathcal{H})|$ . Razred  $\Gamma_{sb}$  hi-pergrafov, ki dosežejo to mejo, je induciran hereditarno. Analiziramo nekaj lastnosti hipergrafov iz razreda  $\Gamma_{sb}$ , pa tudi lastnosti hipergrafov iz razreda prepovedanih hipergrafov za  $\Gamma_{sb}$ . Karakteriziramo vse  $\theta$ -hipergrafe iz razreda  $\Gamma_{sb}$ , od tod pa dobimo tudi karakterizacijo vseh  $\theta$ -hipergrafov, ki so prepovedani za  $\Gamma_{sb}$ .

*Ključne besede: Hipergrafi, vsotno seznamsko barvanje,  $\theta$ -hipergrafi.*

*Math. Subj. Class. (2020): 05C15, 05C65*

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