

A-trails of embedded graphs and twisted duals*

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Abstract

Kotzig showed that every connected 4-regular plane graph has an A -trail—an Eulerian circuit that turns either left or right at each vertex. However, this statement is not true for Eulerian plane graphs and determining if an Eulerian plane graph has an A -trail is NP-hard. The aim of this paper is to give a characterization of Eulerian embedded graphs having an A -trail. Andersen et al. showed the existence of orthogonal pairs of A -trails in checkerboard colourable 4-regular graphs embedded on the plane, torus and projective plane. A problem posed in their paper is to characterize Eulerian embedded graphs (not necessarily checkerboard colourable) which contain two orthogonal A -trails. In this article, we solve this problem in terms of twisted duals. Several related results are also obtained.

Keywords: Embedded graphs, twisted duals, Eulerian, A -trails, checkerboard colourable.

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A-tiri vloženi grafov in zasukanih dualov*

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Povzetek

Kotzig je dokazal, da ima vsak povezan 4-regularen ravninski graf A -tir: Eulerjev obhod, ki v vsakem vozlišču zavije bodisi levo bodisi desno. Ta ugotovitev pa ne drži za Eulerjeve ravninske grafe, in ugotavljanje, ali ima neki Eulerjev ravninski graf A -tir, je NP-težak problem. Cilj tega članka je karakterizirati tiste Eulerjeve vložene grafe, ki imajo A -tir. Andersen in dr. so dokazali obstoj ortogonalnih parov A -tirov v šahovnično pobarvljivih 4-regularnih grafih, vloženi v ravnino, torus in projektivno ravnino. V svojem članku so zastavili problem karakterizacije Eulerjevih vloženi grafov (ne nujno šahovnično pobarvljivih), ki vsebujejo dva pravokotna A -tira. V tem članku ta problem rešimo s pomočjo zasukanih dualov. Na ta način dobimo še nekaj sorodnih rezultatov.

Ključne besede: Vloženi grafi, zasukani duali, Eulerjev obhod, A-tir, šahovnično pobarvljiv.

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