

Point-primitive generalised hexagons and octagons and projective linear groups*

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Abstract

We discuss recent progress on the problem of classifying point-primitive generalised polygons. In the case of generalised hexagons and generalised octagons, this has reduced the problem to primitive actions of almost simple groups of Lie type. To illustrate how the natural geometry of these groups may be used in this study, we show that if \mathcal{S} is a finite thick generalised hexagon or octagon with $\mathcal{G} \leq \text{Aut}(\mathcal{S})$ acting point-primitively and the socle of \mathcal{G} isomorphic to $\text{PSL}_n(q)$ where $n \geq 2$, then the stabiliser of a point acts irreducibly on the natural module. We describe a strategy to prove that such a generalised hexagon or octagon \mathcal{S} does not exist.

Keywords: Generalised hexagon, generalised octagon, generalised polygon, primitive permutation group.

Math. Subj. Class.: 51E12, 20B15, 05B25

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Točkovno primitivni posplošeni šestkotniki in osemkotniki in projektivne linearne grupe*

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Povzetek

Predstavimo nedavni preboj v zvezi s problemom klasificiranja točkovno primitivnih posplošenih mnogokotnikov. V primeru posplošenih šestkotnikov in posplošenih osemkotnikov se na podlagi tega rezultata omenjeni problem poenostavi na obravnavo primitivnih delovanj skoraj enostavnih Liejevih grup. Kot primer, kako se da naravno geometrijo teh grup uporabiti pri tej raziskavi, pokažemo, da če je \mathcal{S} končen odebeljen posplošen šestkotnik ali osemkotnik, na katerem grupa $\mathcal{G} \leq \text{Aut}(\mathcal{S})$ deluje točkovno primitivno, njen podstavek pa je izomorfen $\text{PSL}_n(q)$, kjer je $n \geq 2$, potem stabilizator poljubne točke deluje ireducibilno na naravnem modulu. Opišemo strategijo dokaza, da tak posplošeni šestkotnik oz. osemkotnik \mathcal{S} ne obstaja.

Ključne besede: Posplošeni šestkotnik, posplošeni osemkotnik, posplošeni mnogokotnik, primitivna permutacijska grupa.

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